Measuring neutrinos with Lyman - α observations

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Outline

- Cosmic Calender
- Cosmic Ingredients
- Neutrino in Cosmology
- Lyman - α forest
- Method
- Results
- Summary

Cosmic Calender

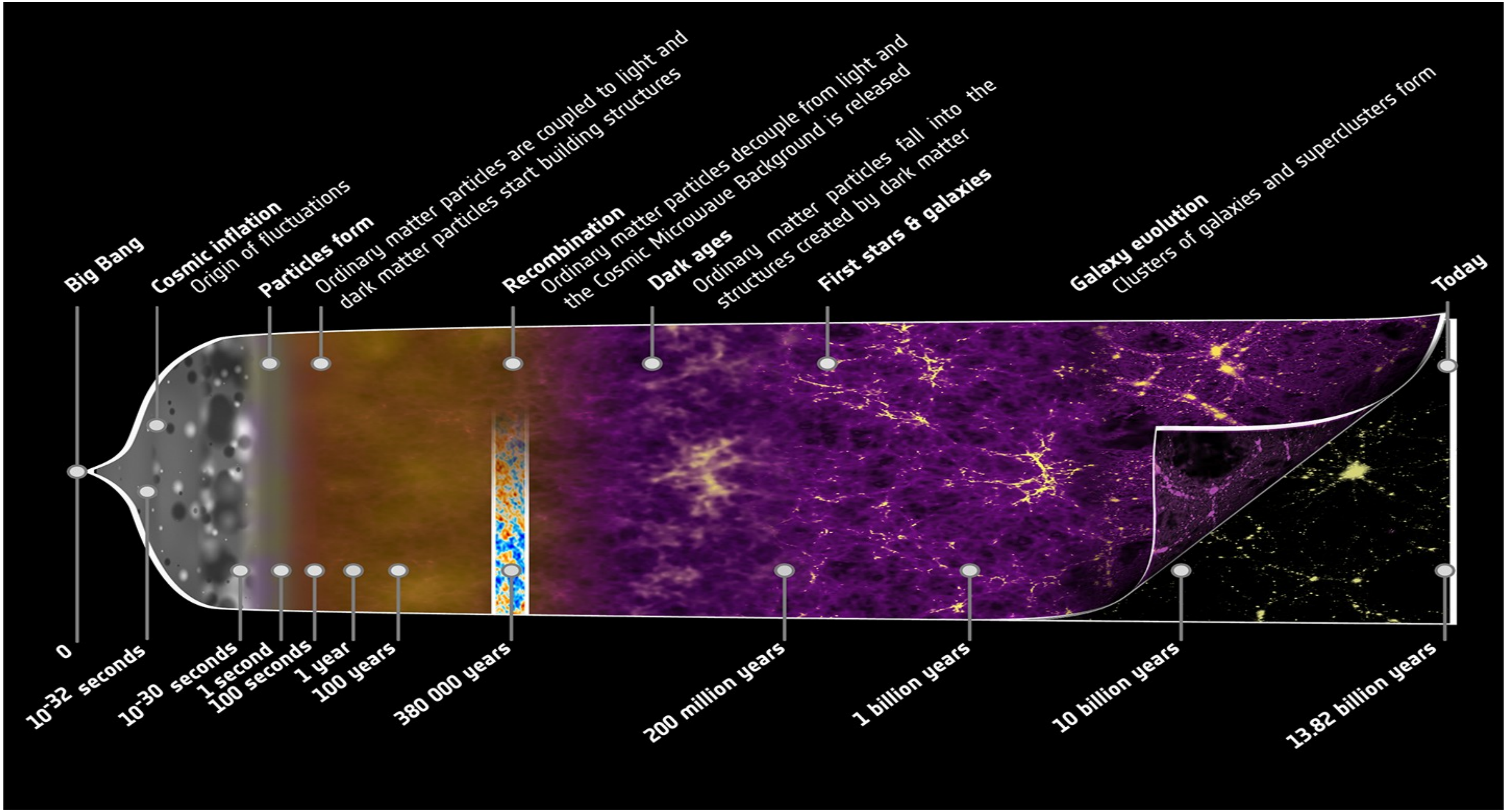
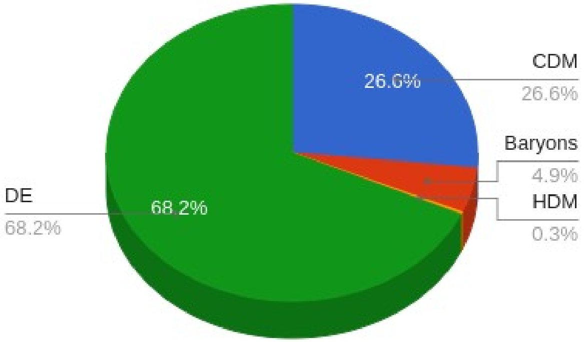


Image credit: google

Cosmic Ingredients



Matter = CDM + Baryons + **Neutrino (HDM)**

Only a tiny fraction of the total matter

$$\Omega_{\nu 0} = 0.0028 (= 0.28\%)$$ [PLANCK 2018]

Contrast with :

$$\Omega_{\rm CDM} = 0.2665 (= 26.65\%)$$

$$\Omega_{\rm b0} = 0.049 (= 4.9\%)$$

Credit : www.rapidtables.com/tools/pie-chart.html

Neutrino as a particle

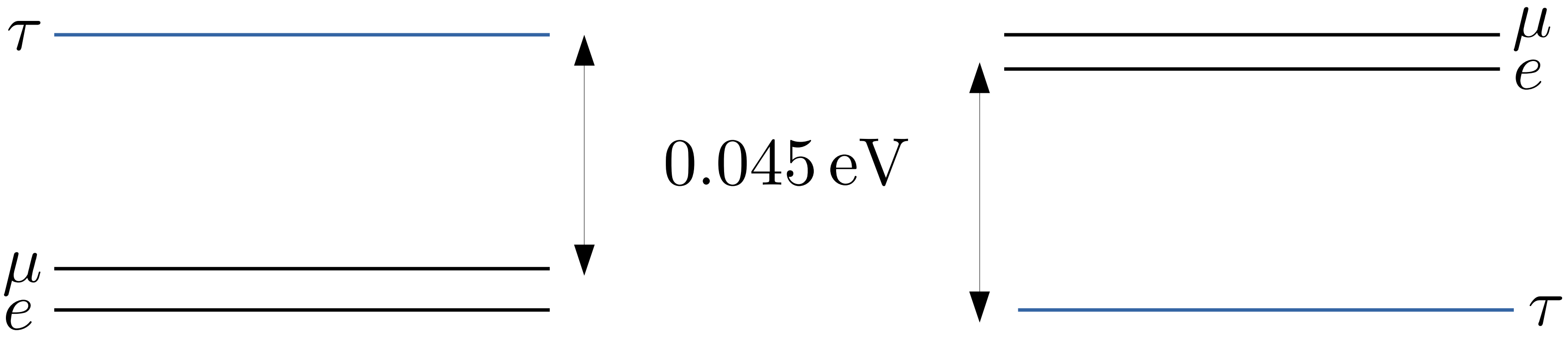
- Neutrino comes in 3 flavours
- Electron neutrino, Mu neutrino & Tau neutrino
- Neutrino oscillation experiments
- Measures the mass squred differences of the neutrinos

$$\Delta m_{32}^2 = 2.43 \pm 0.13 \times 10^{-3}\,\mathrm{eV}^2$$

$$\Delta m_{21}^2 = 7.59 \pm 0.21 \times 10^{-5}\,\mathrm{eV}^2$$

[Kam-LAND 2008]

Neutrino mass hierarchies :



(Normal)   (Inverted)

$$\sum m_\nu > 0.045\,\mathrm{eV} \qquad\qquad \sum m_\nu > 0.09\,\mathrm{eV}$$

Constrain neutrino mass to $\sum m_\nu < 0.09\,\mathrm{eV}$ (rule out inverted hierarchy !!)

Neutrinos in Cosmology

- Neutrinos are relativistic at early times
- Becomes non-relativistic at late times
- Hot Dark Matter (HDM) like component
- Free streams out of the high density regions to the lower density regions, damping the growth of structures at small scales

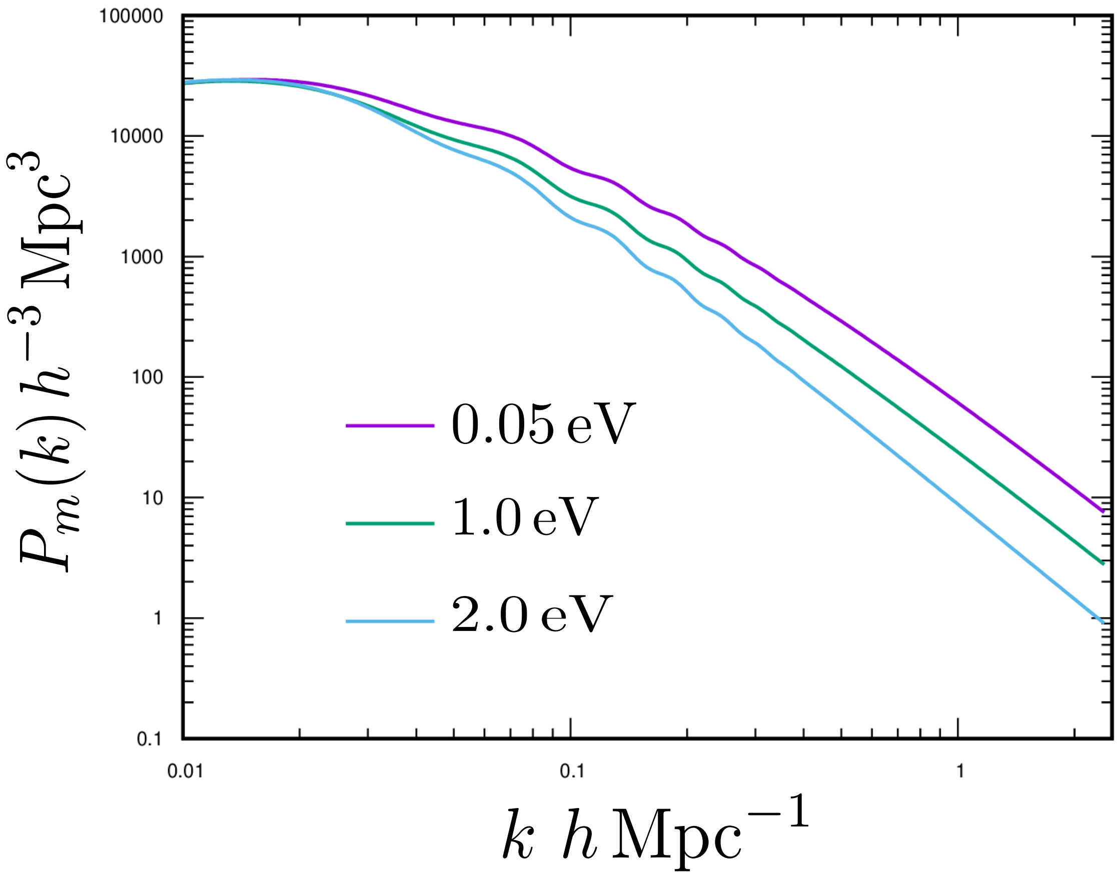
Free streaming scale $k_{\mathrm{fs}} = \sqrt{\frac{3}{2}} \frac{H(z)}{v_{\mathrm{th}}(1+z)}$

e.g. for a matter dominated universe, $k_{\mathrm{fs}} \propto (1+z)^{-1/2}$

Matter perturbations are suppressed at scales $k > k_{\mathrm{fs}}$

[Agarwal & Feldman 2011]

[Matter Power Spectrum + Neutrino]



- More is the mass, more will be the suppression
- Suppression is more at smaller scales (at large k)

[Credit : CAMB Online]

Cosmological probes of small scales

- Galaxy Surveys : Probe the scales $k \sim 0.2h\,\mathrm{Mpc}^{-1}$
- CMB & Weak Lensing Surveys : Probe scales upto $k \sim 0.3h\,\mathrm{Mpc}^{-1}$
- Lyman – α Forest : Probe scales as low as $k \sim 5h\,\mathrm{Mpc}^{-1}$

  an excellent probe of matter power at scales $< 1\,\mathrm{Mpc}$!!

[Tegmark 2002]

Lyman – α data alone :

$$\sum m_\nu < 0.71\,\mathrm{eV}$$

Lyman - α data + CMB + Lensing + BAO :

$$\sum m_\nu < 0.11\,\mathrm{eV} \quad (95\%\,\mathrm{CL})$$

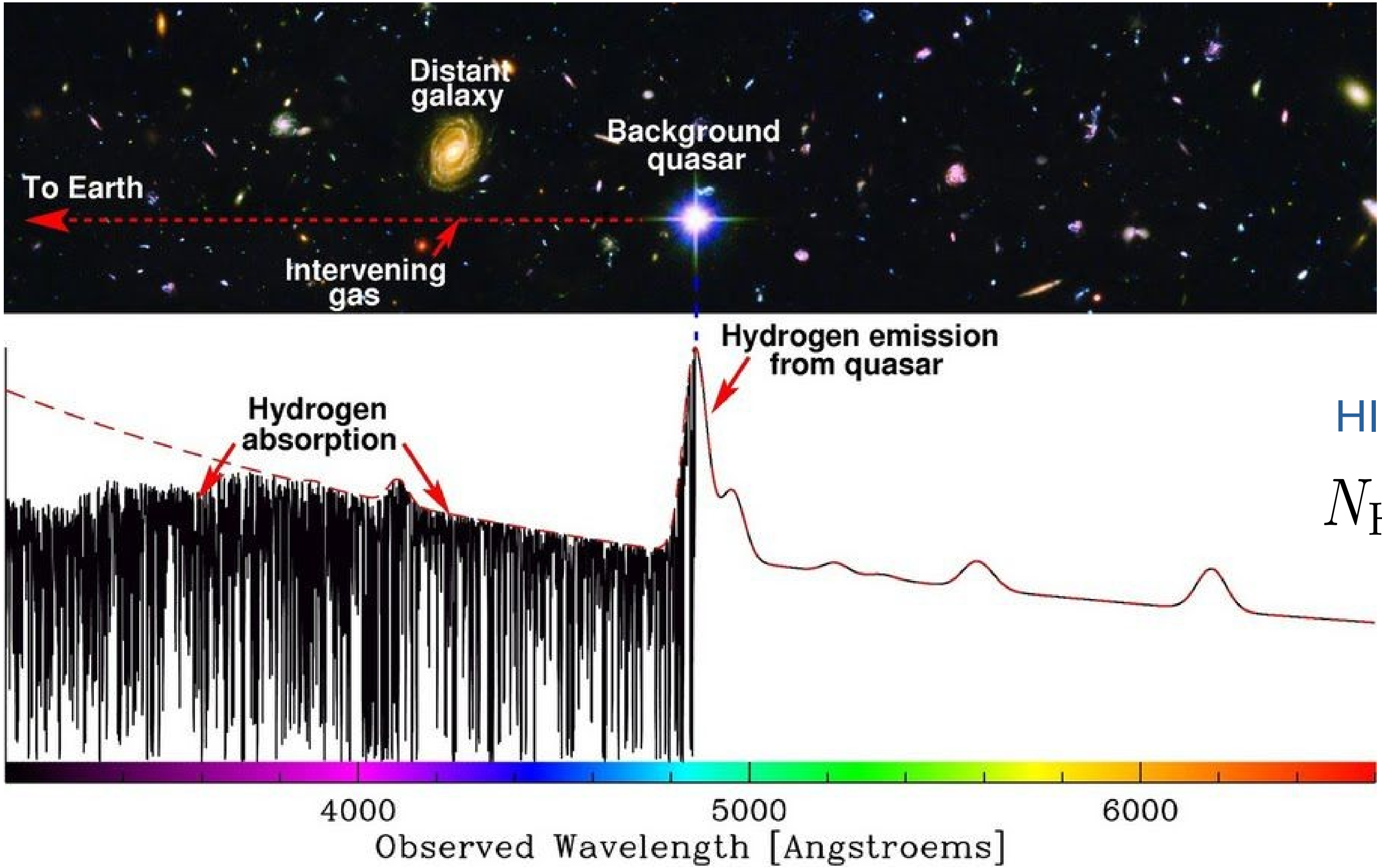
[Palanque -Delabrouille 2019]

CMB + Polarization + Supernovae 1a data + BAO measurements + growth parameter :

$$\sum m_\nu < 0.09\,\mathrm{eV} \quad (95\%\,\mathrm{CL})$$

[Valentino et al. 2021]

Lyman-α forest



HI fraction

$$x_{\mathrm{HI}} \leq 10^{-4}$$

HI Column number density

$$N_{\mathrm{HI}} = 10^{12} - 10^{15}\,\mathrm{cm}^{-2}$$

Image credit: google

Observables of Lyman-α forest

Observed Flux Contrast :

$$\delta_F = \frac{F - \bar{F}}{\bar{F}}$$

F : Transmitted flux

$\bar{F}$: Unabosrbed quasar continnum

$$F = \bar{F}\, e^{-\tau}$$

τ : Lyman - α optical depth

$$\delta_F = \left(e^{-\tau} - 1\right)$$

Lyman - α optical depth:

$$\tau = A(J_0, T_0) \times \left(\frac{n_b}{n_0}\right)^{[\,2-0.7(\gamma-1)=\beta\,]}$$

[Choudhury et al. (2001), Pandey & Sethi (2012)]

$$A(z) = 0.946 \left(\frac{1+z}{4}\right)^6 \left(\frac{\Omega_b h^2}{0.022}\right)^2 \left(\frac{T_0}{10^4\mathrm{K}}\right)^{-0.7} \left(\frac{J}{10^{1}2\mathrm{s}^{-1}}\right)^{-1} \left(\frac{H(z)}{H_0}\right)^{-1}$$

Lyman - α modelling parameters :

J_0 : Photoionization rate [ionization state]

T_0 : IGM temperature [thermal state]

γ : Semi - adiabatic index for the Lyman - α clouds [dynamical state]

[Hui & Gnedin 1997]

Log-normal approximation :

$$n_b \propto e^{\delta_b}$$

Lyman – α optical depth :

$$\tau(z) = A(z) \times e^{\beta(\delta_b - \langle \delta_b^2 \rangle /2)}$$

$$\langle \delta_b^2 \rangle = \xi(\Delta z)\big|_{\Delta z = 0}$$ [2-point correlation function at zero lag]

Computing 2-point correlation function

Begin with the matter power spectrum : $P_m(k, z)$

Use the Jean's cut to get the baryon power spectrum :

$$P_b(k) = \frac{P_m(k, z)}{[1 + (k/k_{\rm J})^2]^2}$$

$$k_{\rm J} \simeq 5\,{\rm Mpc}^{-1} \text{ at } z = 2 \;(\lambda_{\rm J} = 0.2\,{\rm Mpc})$$

Compute 1D Baryon power spectrum :

$$P_{1D}(k_{1D}, z) = \frac{1}{2\pi} \int_{|k_{1D}|}^{\infty} P_b(k^{'}, z)\, k^{'} dk^{'}$$

Use the above 1D baryon power spectrum to compute :

$$\xi(\Delta z) = \frac{1}{2\pi r_z^{'}} \int_{-\infty}^{\infty} P_{1D}(k_{1D}, z) e^{ik_{1D}\Delta z} dk_{1D},$$

gives the 2-point correlation function $\xi(\Delta z)$ at a given Δz

Recall that :

$$\delta_F = \left(e^{-\tau} - 1\right)$$

For optically thin IGM:

$$\delta_F = -\tau \qquad (\tau << 1)$$

Compute the mean:

$$\langle\delta_F\rangle = -\langle\tau\rangle \;\; = -\left[A \times e^{(\beta^2-\beta)\xi(0)/2}\right]$$

Redshift z = 2

$$\langle\delta_F\rangle \simeq -0.11$$

Building the estimator:

$$\hat{E}_{FF}(z_a, z_b) = \underbrace{\delta_F(z_a)\,\delta_F(z_b)}_{\text{Lyman}-\alpha\text{ forest}} + \underbrace{\delta_N(z_a)\,\delta_N(z_b)}_{\text{Noise}}$$

$$= \hat{C}_{FF}(z_a, z_b) + \hat{C}_{NN}(z_a, z_b)$$

$$\langle \hat{C}_{FF}(z_a, z_b) \rangle = C_{FF}(z_a, z_b) = A^2 \times e^{(\beta^2 - \beta)\xi(0)} \times e^{\beta^2 \xi(\Delta z_{ab})}$$

$$\langle \hat{C}_{NN}(z_a, z_b) \rangle = \sigma_N^2 \, \delta_{a,b}$$

Noise in 2 different redshifts are uncorrelated

Considering multiple redshift pairs:

$$\hat{E}_{FF}(z_a, z_b) = \frac{1}{n_{\rm pair}} \sum_{a=1}^{n_{\rm pair}} \delta_F(z_a)\,\delta_F(z_b) + \delta_N(z_a)\,\delta_N(z_b)$$

Mean of the estimator:

$$\langle\, \hat{E}_{FF}(z_a, z_b)\,\rangle = C_{FF}(z_a, z_b) + \sigma_N^2\,\delta_{a,b}$$

Covariance of the estimator:

$$\left\langle \left[\hat{E}_{FF}(z_a, z_b) - E_{FF}(z_a, z_b) \right] \left[\hat{E}_{FF}(z_c, z_d) - E_{FF}(z_c, z_d) \right] \right\rangle$$

$$= A^4 \times e^{2(\beta^2 - \beta)\xi(0)} \times e^{\beta^2 (\xi_{ab} + \xi_{cd})}$$

$$\times \left[e^{\beta^2 (\xi_{ac} + \xi_{ad} + \xi_{bd} + \xi_{bc})} - 1 \right]$$

$$+ \frac{1}{n_{\text{pair}}^2} \sum_{a,c=1}^{n_{\text{pair}}} \left[\sigma_N^4 \left(\delta_{ac} \delta_{bd} + \delta_{ad} \delta_{bc} \right) \right]$$

“Zero-Noise” Estimator:

$$\left\langle \left[\hat{E}_{FF}(z_a, z_b) - E_{FF}(z_a, z_b) \right] \left[\hat{E}_{FF}(z_c, z_d) - E_{FF}(z_c, z_d) \right] \right\rangle$$

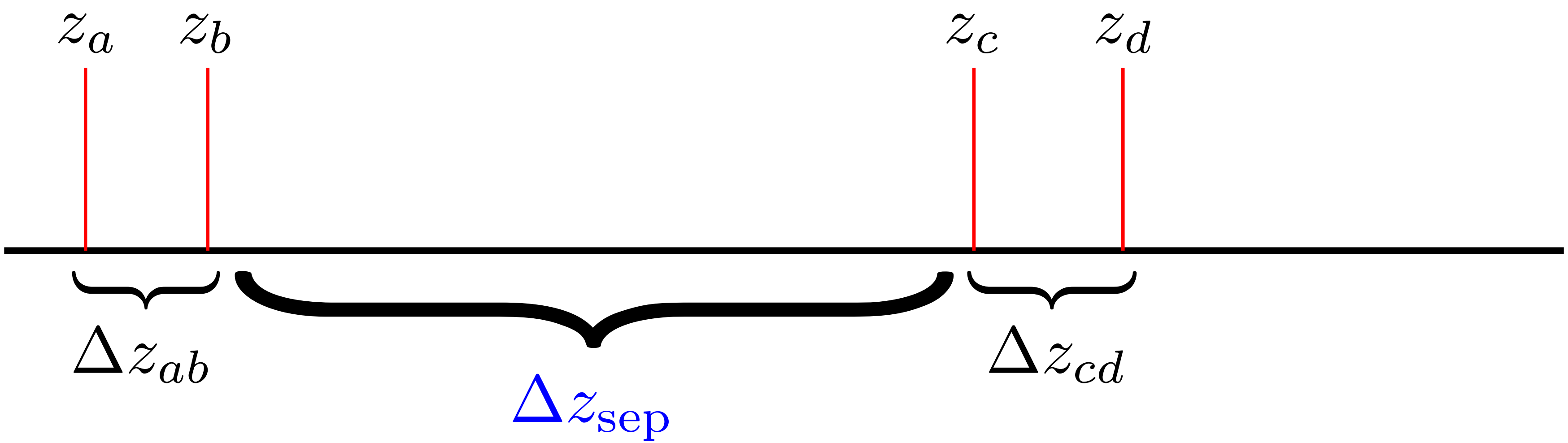
$$= \mathrm{COV}(z_a, z_b; z_c, z_d) \equiv \mathrm{COV}(\Delta z_m; \Delta z_n)$$

$$= A^4 \times e^{2(\beta^2-\beta)\xi(0)} \times e^{\beta^2(\xi_{ab}+\xi_{cd})}$$

$$\times \left[e^{\beta^2(\xi_{ac}+\xi_{ad}+\xi_{bd}+\xi_{bc})} - 1 \right]$$

Note: Redshift pairs are well separated from each other !!

Schematic of the Lyman - α observation:



Total redshift strech covered: $\Delta z + \Delta z_{\text{sep}} + \Delta z$

$$\Delta z_{\text{sep}} = [\,0.04 - 0.06\,] \quad (\,\Delta r_{\text{sep}} \; = \; 40\text{ - }60\,\text{Mpc}\,)$$

$$\Delta z_{ab},\; \Delta z_{cd} \;\leq\; 0.02 \qquad (\,\Delta r_{ab}, \Delta r_{cd} \;=\; 20\,\text{Mpc}\,)$$

$$\text{COV}(\Delta z_m,\; \Delta z_n) = \sigma^2_{\text{COV}}\; \delta_{m,n}$$

$$\sigma^2_{\text{COV}} \;=\; [\,10^{-5}\text{ - }10^{-7}\,]$$

Fisher matrix:

$$F_{pq} = \frac{1}{\sigma^2_{\mathrm{COV}}} \sum_m \frac{\partial C_{FF}(\Delta z_m)}{\partial \lambda_p} \frac{\partial C_{FF}(\Delta z_m)}{\partial \lambda_q}$$

$$\lambda \equiv [\, \lambda_1,\, \lambda_2,\, ..\, \lambda_N \,]$$

[Parameter Set]

$$\mathrm{SNR}_p = \frac{\bar{\lambda}_p}{\sqrt{[F]^{-1}_{pp}}}$$

[Signal - to - Noise Ratio (SNR)]

Lyman - α Data:

Thermal velocity width: $v_{\rm th} \simeq 10\,{\rm km\,s^{-1}}$

Velocity resolution of the data: $v_{\rm res} = 5\,{\rm km\,s^{-1}}$

$$\Delta r_{\rm res} \simeq 0.05\,{\rm Mpc} \quad (z=2)$$

$$\Delta r_{\rm Jeans} \equiv k_{\rm Jeans}^{-1} \simeq 0.2\,{\rm Mpc}$$

Observing comoving coverage:

$$3 \leq \Delta r \leq 20\,\mathrm{Mpc} \quad (\Delta r_{\mathrm{interval}} = 0.2\,\mathrm{Mpc})$$

$$\Delta z_{\mathrm{stretch}} = 2 \times \Delta z + \Delta z_{\mathrm{sep}} (= 0.06)$$
$$= 0.1$$

Lyman - α observation in the redshift range:

$$2.003 \leq z \leq 2.1 \quad (\Delta z_{\mathrm{interval}} = 0.0002)$$

Results: $\left[\,\sigma^2_{\mathrm{COV}} = 10^{-5}\,\right]$

1 - parameter case: $\Omega_{\nu 0}h^2 = 0.001 \; \left(\sum m_\nu = 0.1\,\mathrm{eV}\right)$

$$\mathrm{SNR} = 7.5$$

3 - parameter case: $\left[\,\Omega_{\nu 0}h^2,\; \Omega_{m0}h^2,\; n_s\,\right]$

$$\left.\begin{aligned} \Omega_{m0}h^2 &= 0.142 \pm 0.00087\,(\sigma_m) \\ n_s &= 0.9665 \pm 0.0038\,(\sigma_s) \end{aligned}\right\}$$

[Priors from PLANCK 2018]

$$\mathrm{SNR} = 7.2$$

6 - parameter case:

$$\left[\,\Omega_{\nu 0}h^2,\ \Omega_{m0}h^2,\ n_s\ J_0,\ T_0,\ \beta\,\right]$$

$$J_0 = 1.5\times 10^{12}\,\mathrm{s}^{-1}$$

$$T_0 = 0.95\times 10^{4}\,\mathrm{K}$$

$$\beta = 2.21\ (\,\gamma\ =\ 0.7\,)$$

[fiducial values of the Lyman - α nuisance parameters]

[Palanque-Delabrouille 2019]

$$\mathrm{SNR}\ =\ 0.4$$

[a factor of 20 less !!]

Priors on: $T_0,\ \beta$

$$T_0 = 0.95 \pm 0.18\,(\,\sigma_T\,)\ (\,\times 10^4\,\mathrm{K}\,)$$

$$\beta = 2.21 \pm 0.07\,(\,\sigma_\beta\,)$$

Lyman - α observations in the redshift range:

$$2 \leq z \leq 2.1$$

$$\sigma^2_{\mathrm{COV}} = 10^{-7}\ :$$

$$\mathrm{SNR} = \begin{Bmatrix} 1.1 \\ 2.1 \end{Bmatrix} \qquad \sum m_\nu = \begin{Bmatrix} 0.05\,\mathrm{eV} \\ 0.10\,\mathrm{eV} \end{Bmatrix}$$

Conside Lyman - α observations in the redshift range: $2 \leq z \leq 2.5$

$$\Delta z_{\rm sep} = 0.5 - 2 \times 0.02 = 0.41$$

$$(\Delta r_{\rm sep} \simeq 400\,{\rm Mpc})$$

$$\sigma^2_{\rm COV} = 10^{-8} \; :$$

$$\mathrm{SNR} = \begin{Bmatrix} 1.1 \\ 2.1 \end{Bmatrix} \sum m_\nu = \begin{Bmatrix} 0.05\,{\rm eV} \\ 0.10\,{\rm eV} \end{Bmatrix}$$

[3 - 6 σ measurement !!]

Summary

- Neutrinos are Hot Dark Matter - like component of the matter in the universe, Contributing only a tiny fraction (~ 0.28 %) of the matter - energy budget of the universe.

- Neutrinos come in three flavors - electron neutrino, mu neutrino and tau neutrino, giving rise to two mass hierarchies - Normal and Inverted.

- Neutrino causes suppression in the matter power spectrum at small scales; higher the neutrino mass, larger is the suppression.

- Lyman-α forest provides the strongest probe of matter power at small scales, can be used to constrain the neutrino mass.

- Using "Zero - Noise estimator" for the Lyman - α forest, it is possible to have measurement of the neutrinos with masses in the range: [0.05 - 0.1 eV] at [3 - 6 σ] confidence interval.