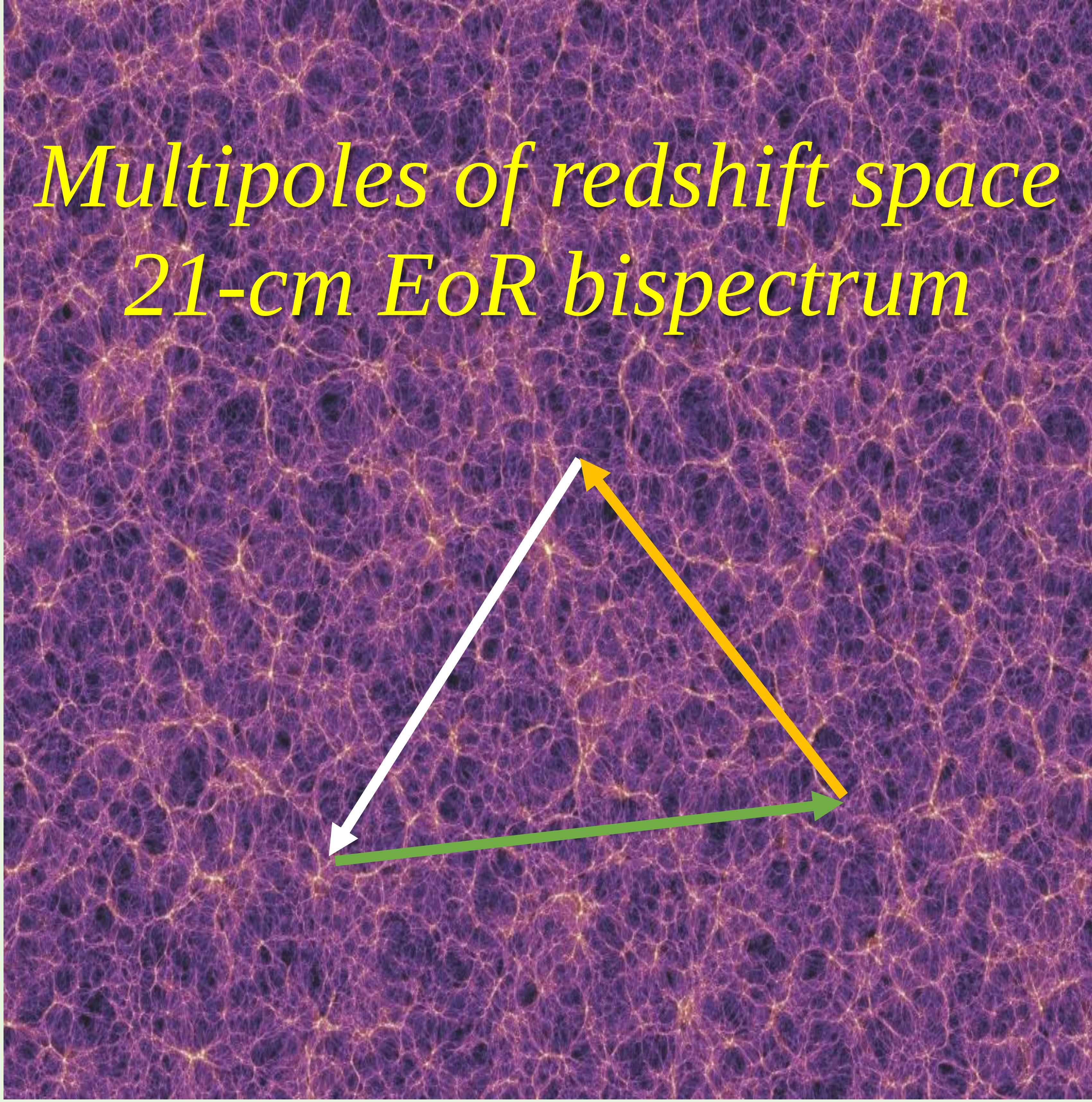
Workshop on 21-cm cosmology 2023
NISER, Bhubaneswar

Multipoles of redshift space 21-cm EoR bispectrum



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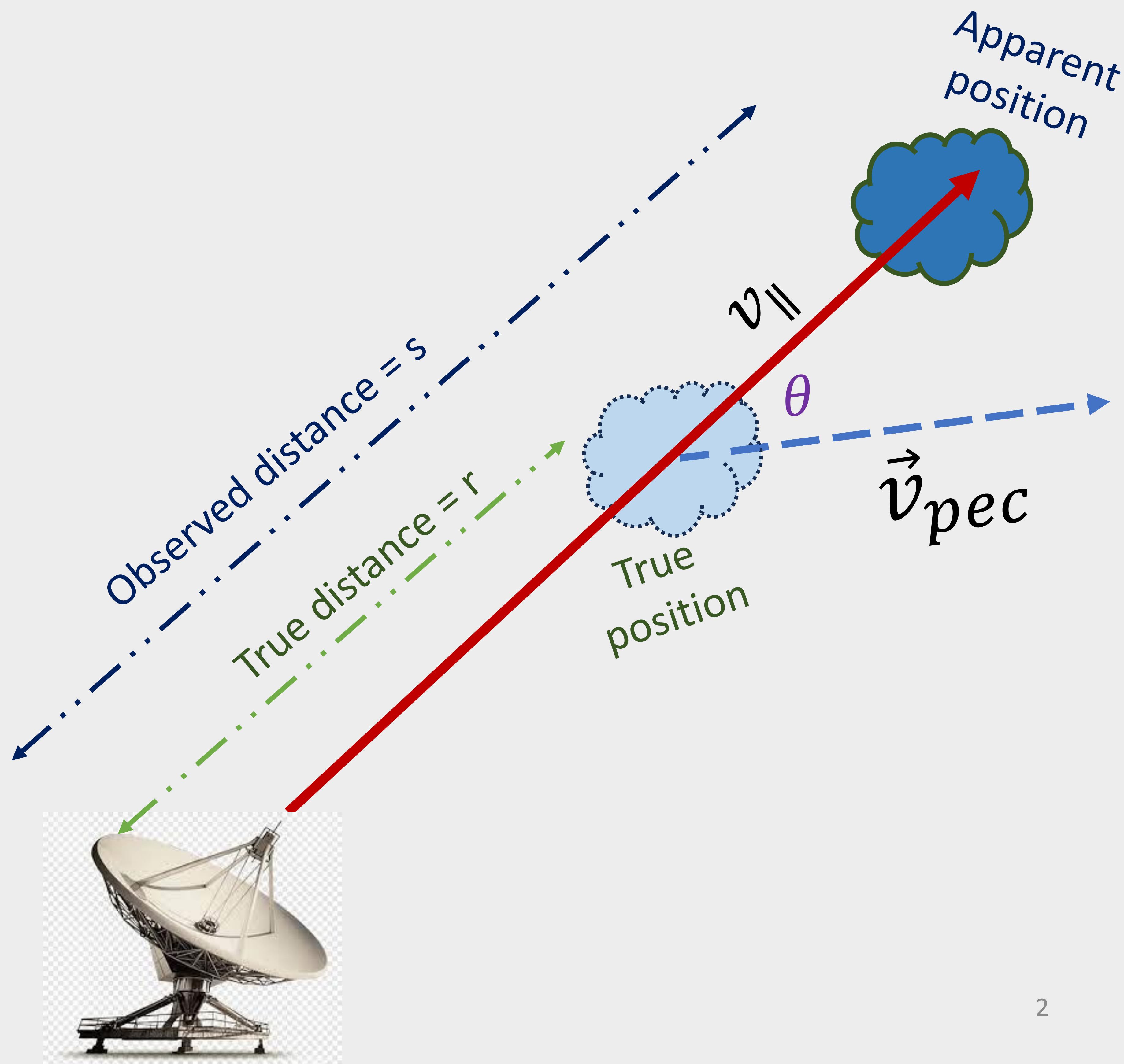
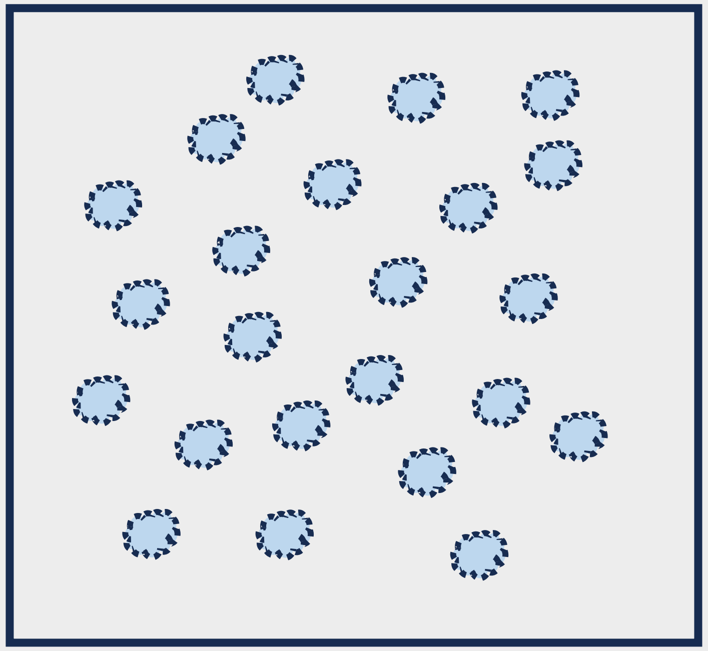
Collaborators: *Suman Pramanick, Somnath Bharadwaj, Abhinash Kumar Shaw & Suman Majumdar*

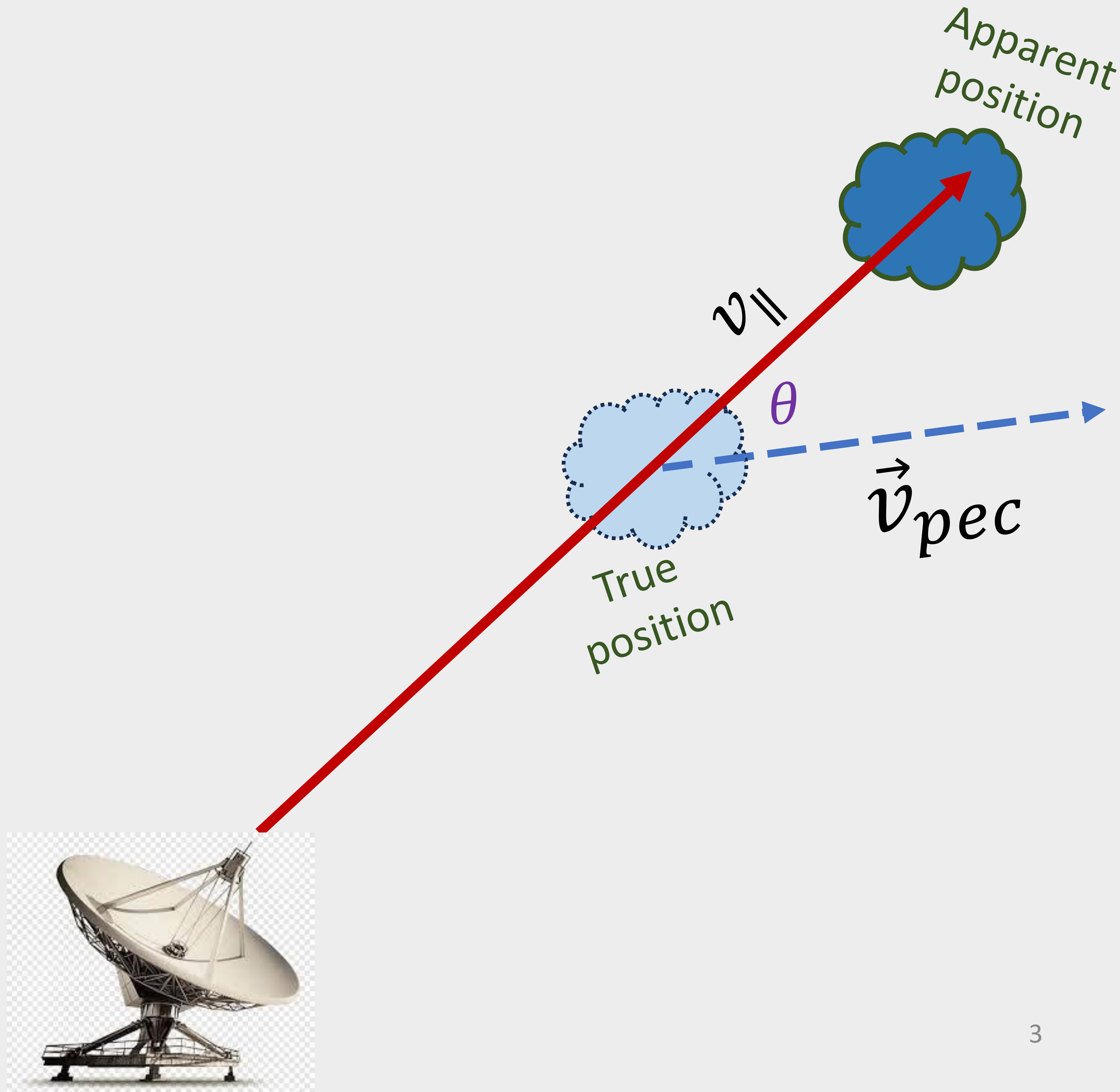
Redshift space distortion



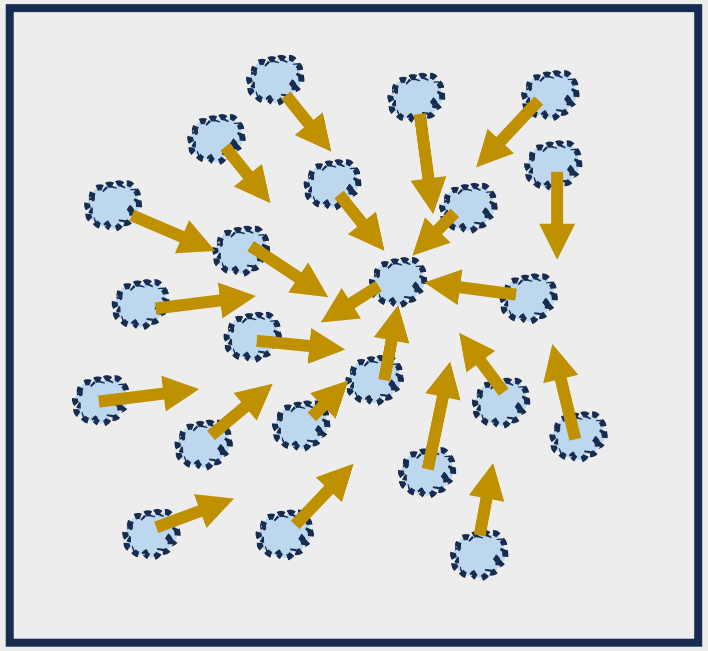
Redshift space distortion



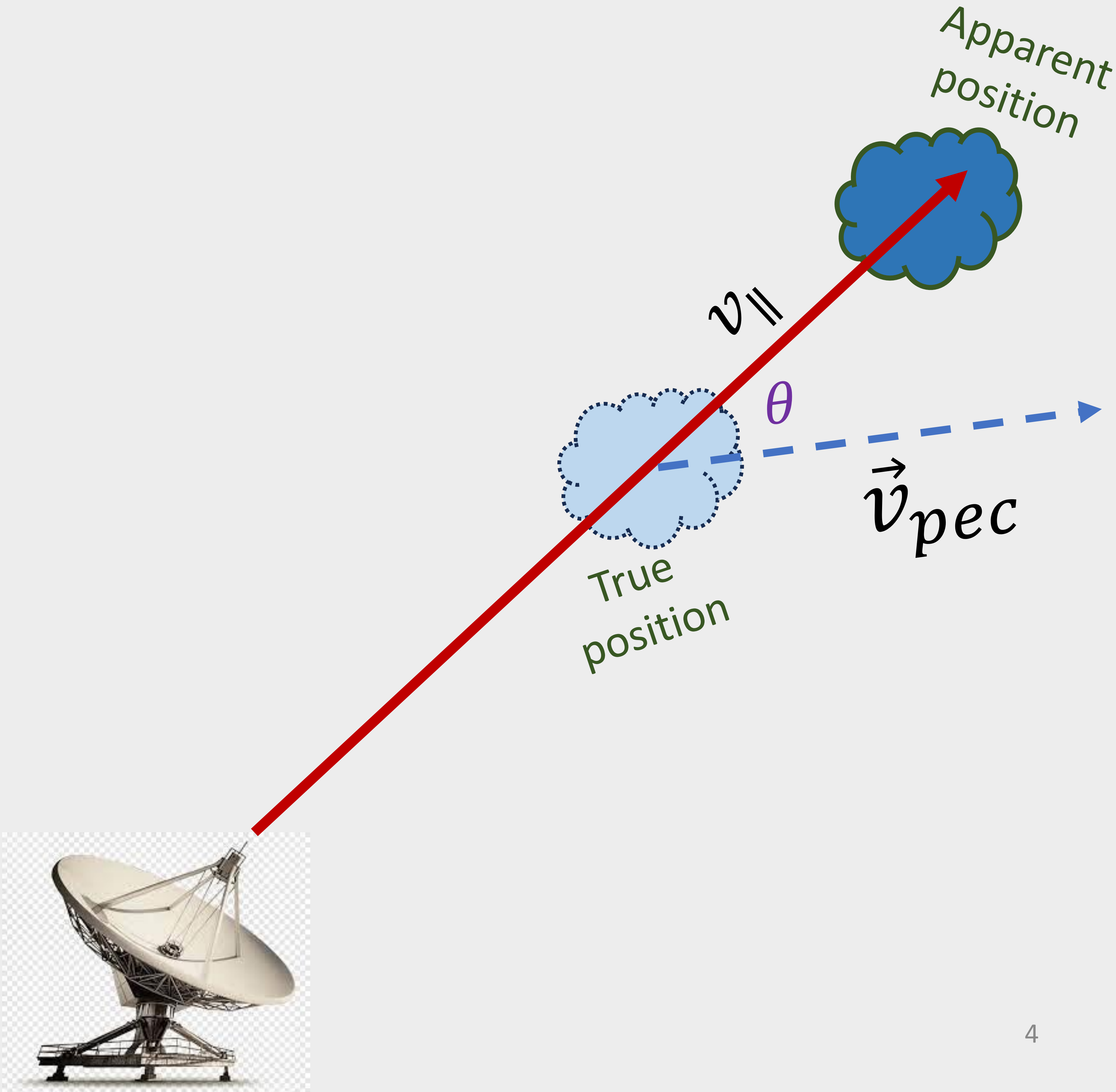
True positions
(Real space)



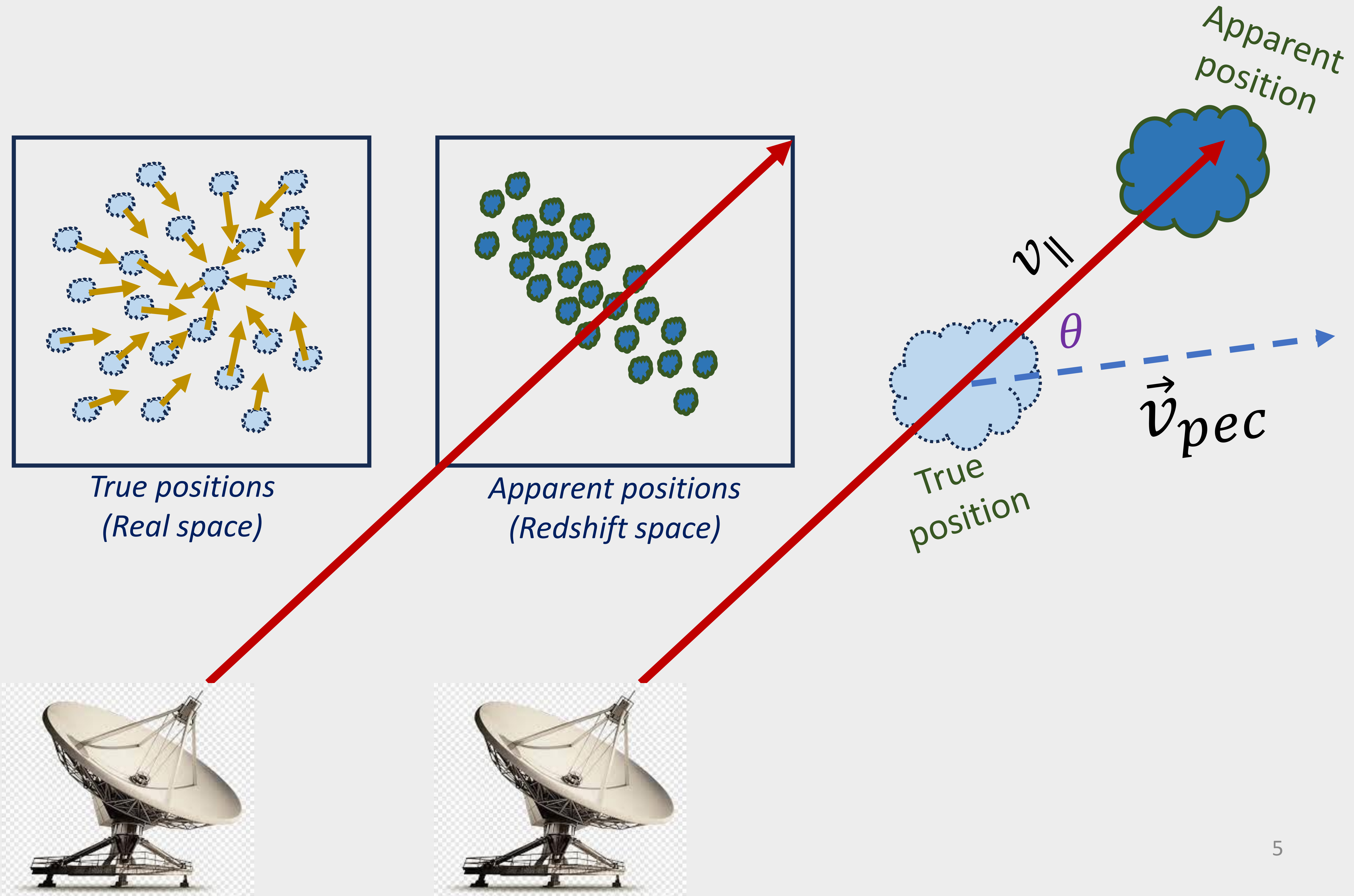
Redshift space distortion

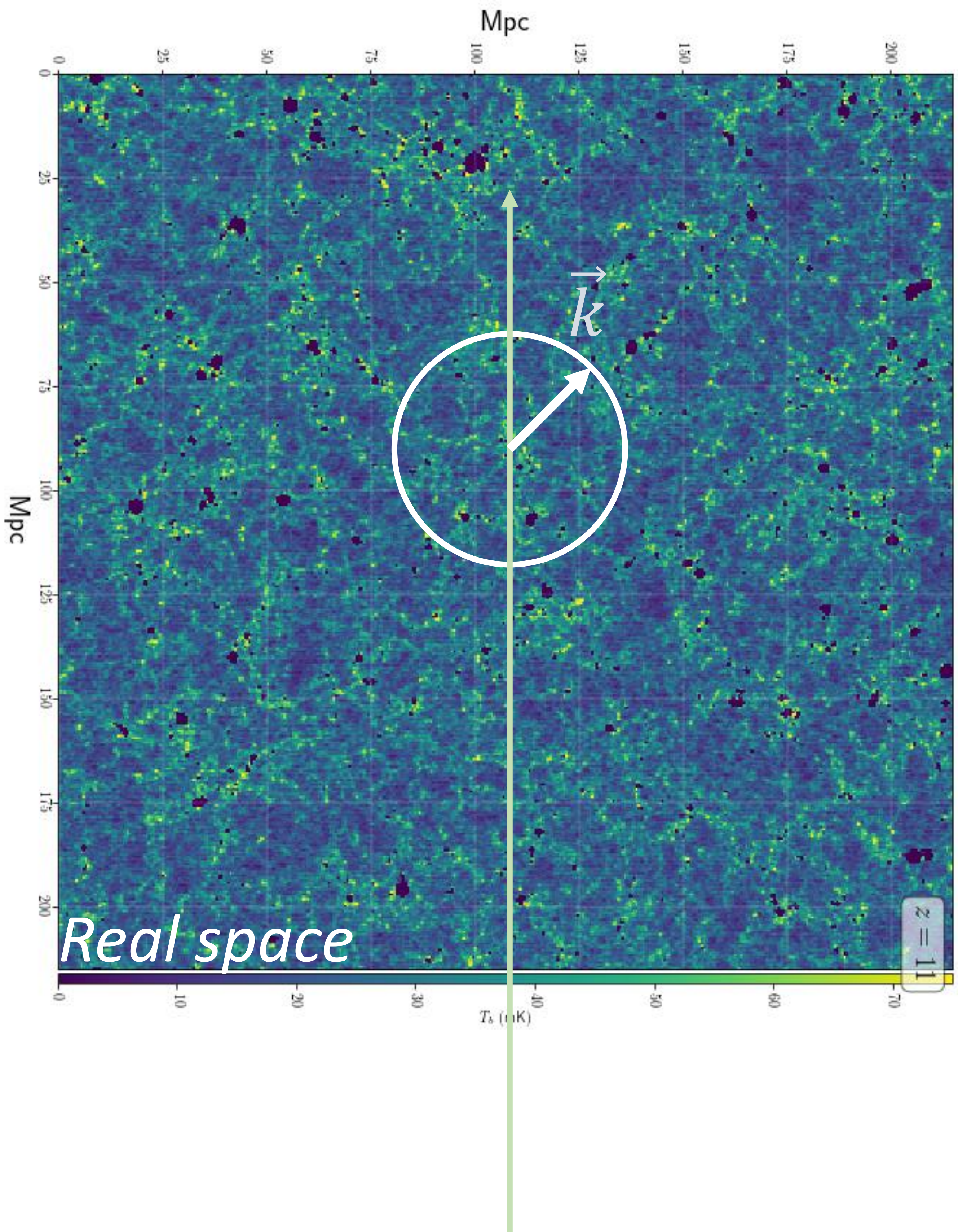


True positions
(Real space)

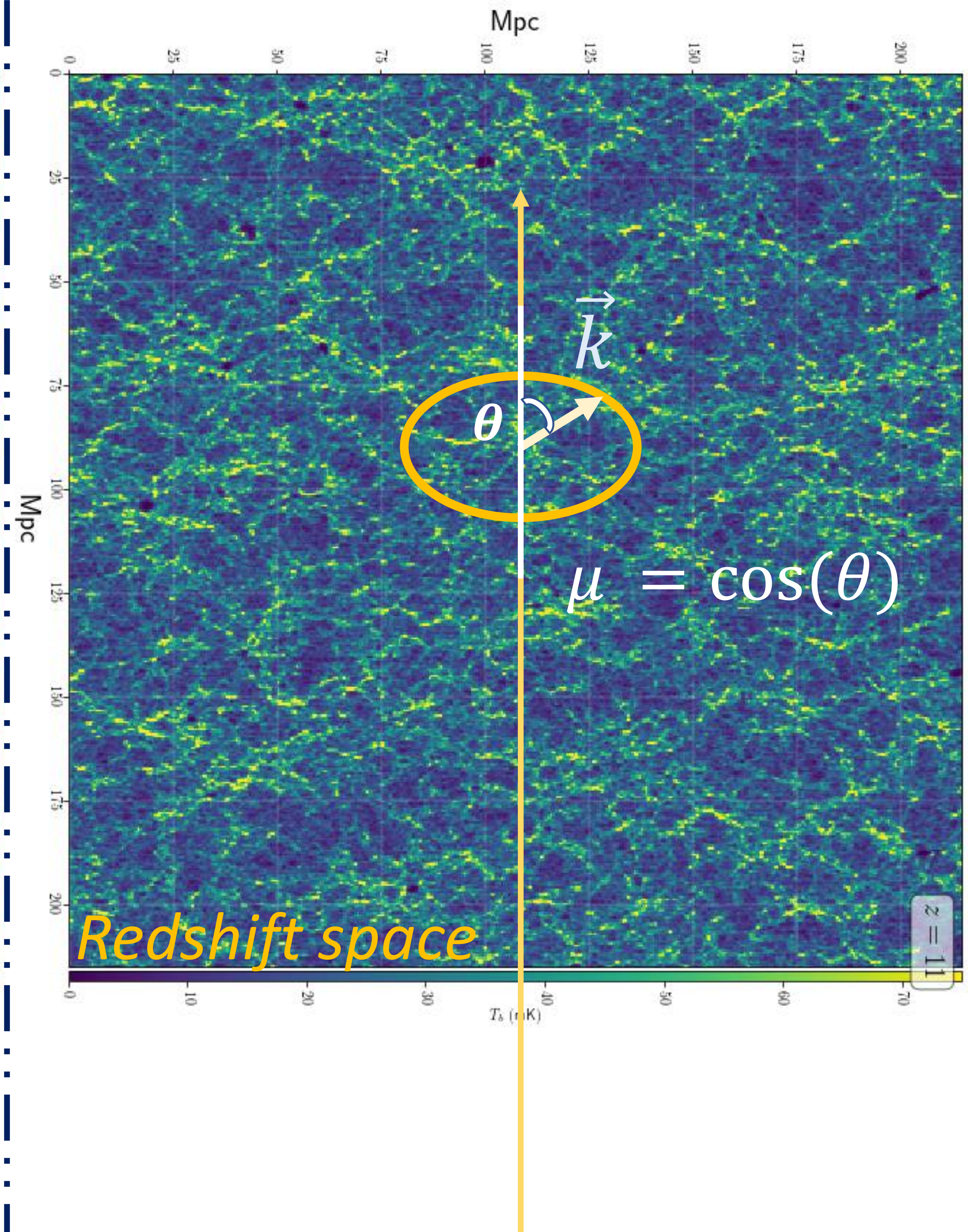


Redshift space distortion





Observer

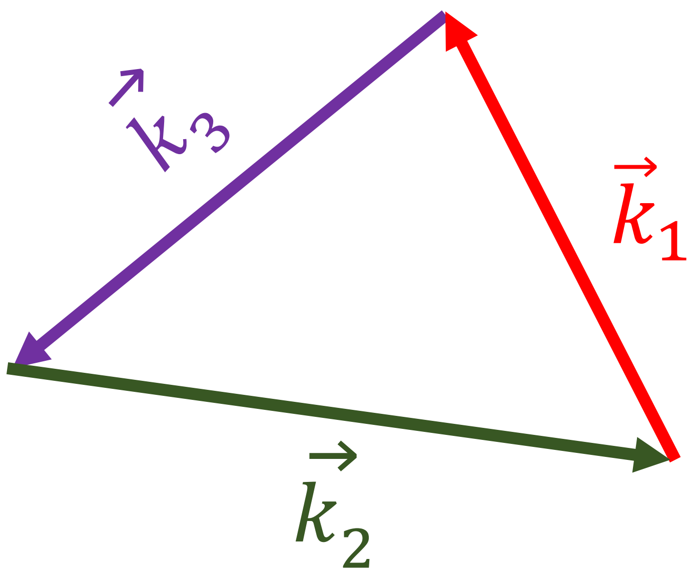


Observer

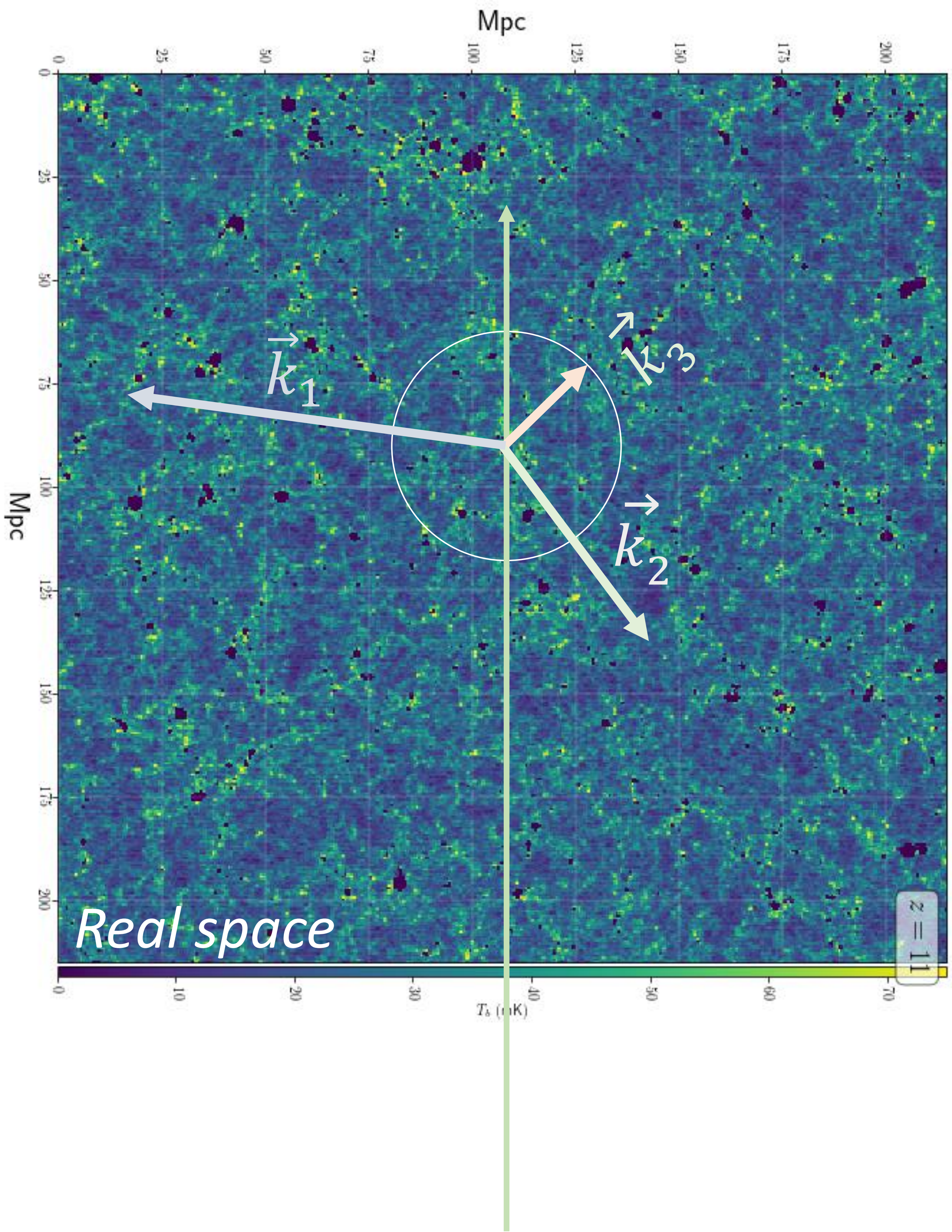
Bispectrum

Lowest order statistics sensitive to non-Gaussianity

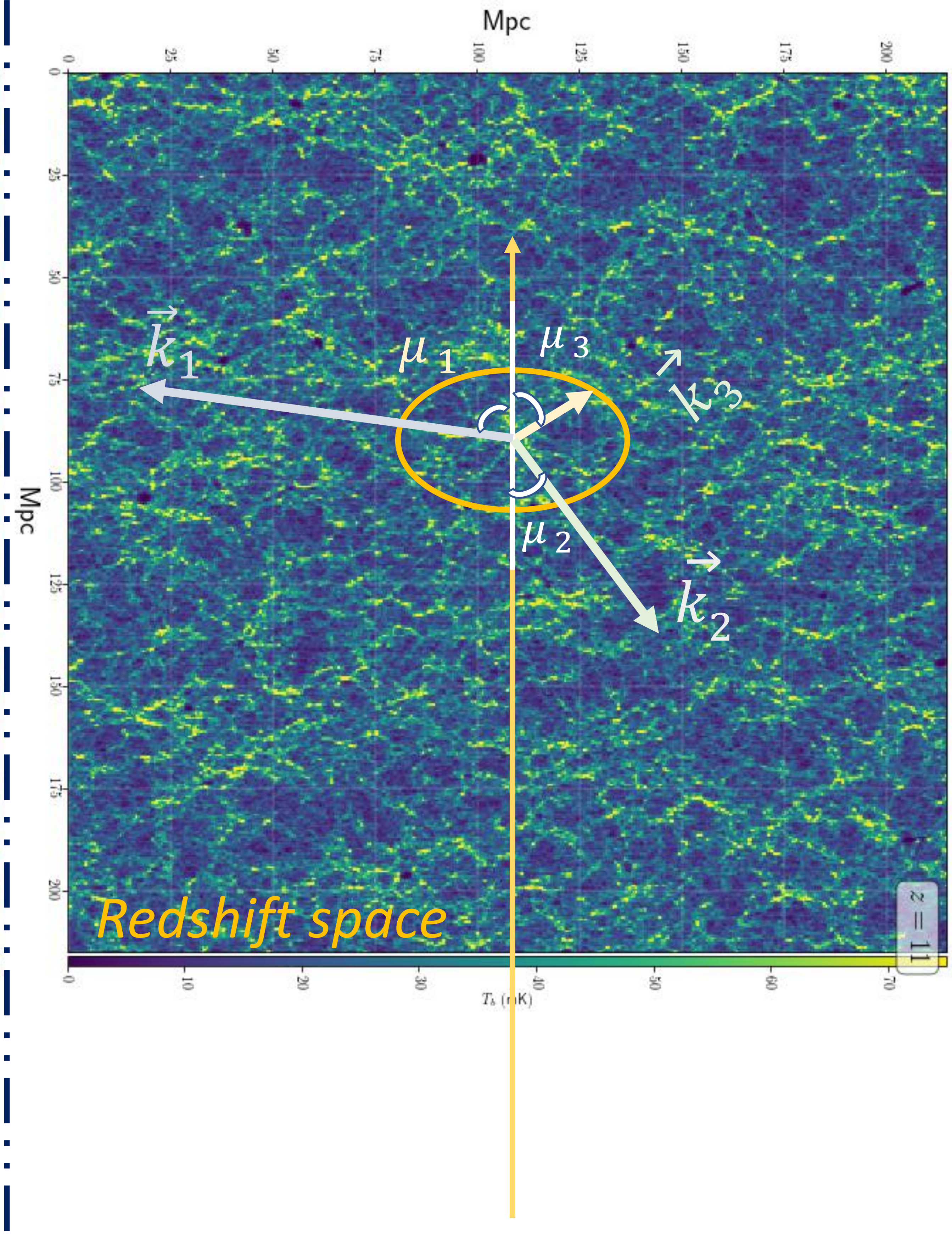
$$B\left(\vec{k}_1, \vec{k}_2, \vec{k}_3\right) \equiv \langle \Delta_{\vec{k}_1} \Delta_{\vec{k}_2} \Delta_{\vec{k}_3} \rangle$$



$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$



Observer

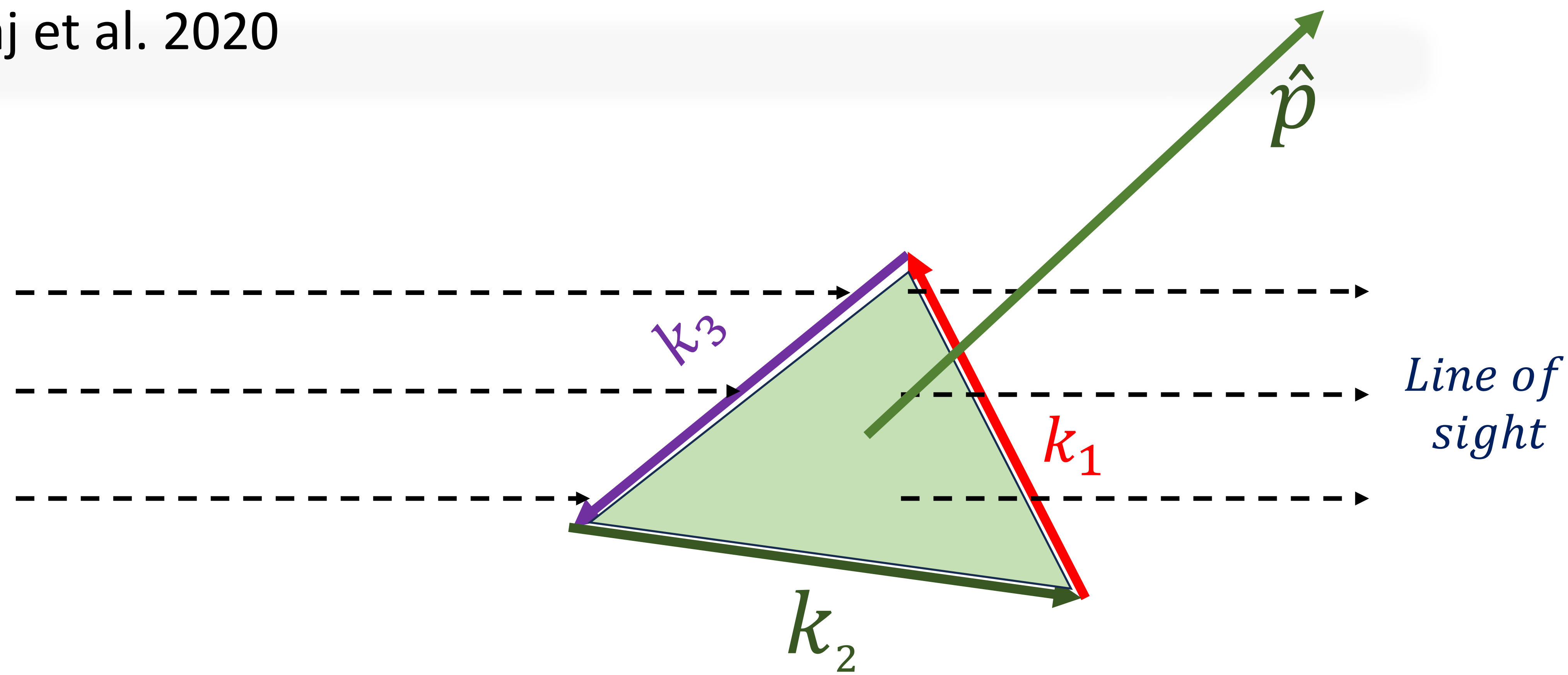


Observer

Multipole Expansion

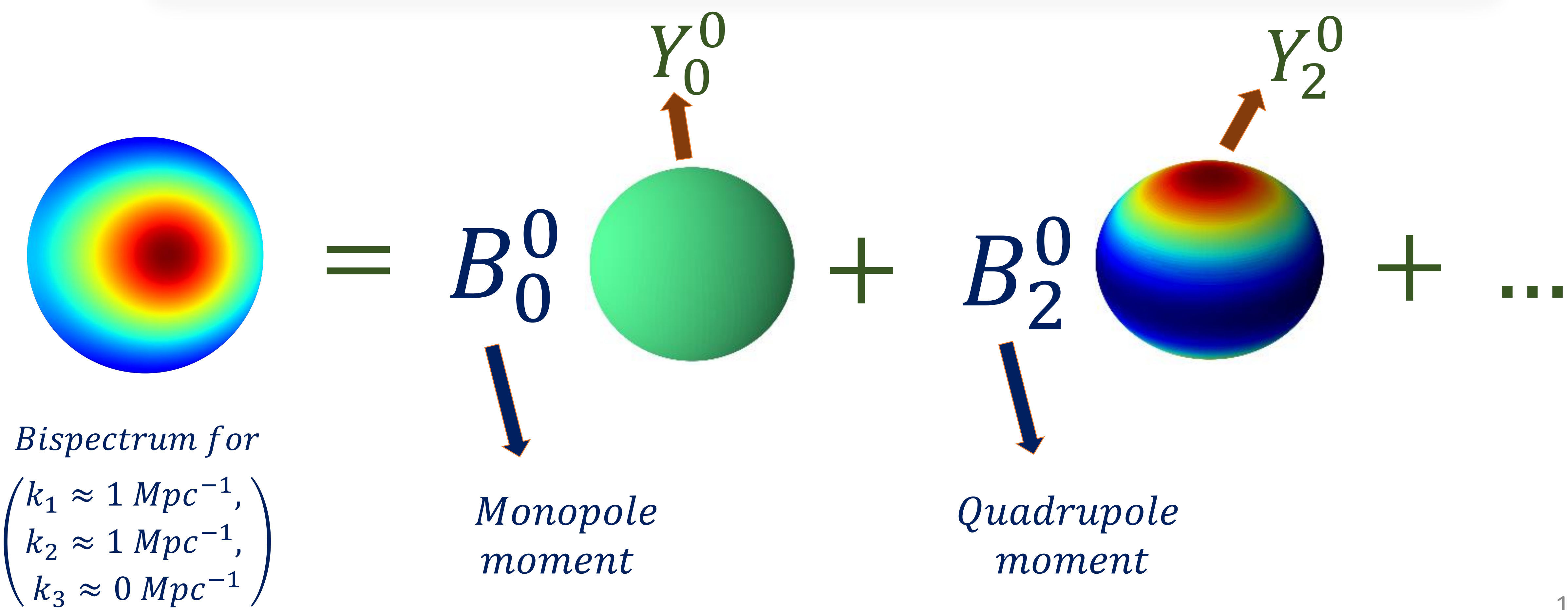
$$B_l^m(k_1, k_2, k_3) = \sqrt{\frac{2l+1}{4\pi}} \int [\mathrm{Y}_l^{\mathrm{m}}(\hat{p})]^* \, B^s(\hat{p}, k_1, k_2, k_3) \, d\Omega_{\hat{p}}$$

Bharadwaj et al. 2020



Multipole Expansion

$$B_l^m(k_1, k_2, k_3) = \sqrt{\frac{2l+1}{4\pi}} \int [\mathrm{Y}_l^m(\hat{p})]^* \, B^s(\hat{p}, k_1, k_2, k_3) \, d\Omega_{\hat{p}}$$



Simulating EoR (ReionYuga)

- Cube vol $= [215\ Mpc]^3$ = $[384$ grids$]^3$
- $M_{min} = 1.09 \times 10^9\ M_{\odot}$
- $N_{ion} = 23.21$
- $R_{mfp} = 20\ Mpc$

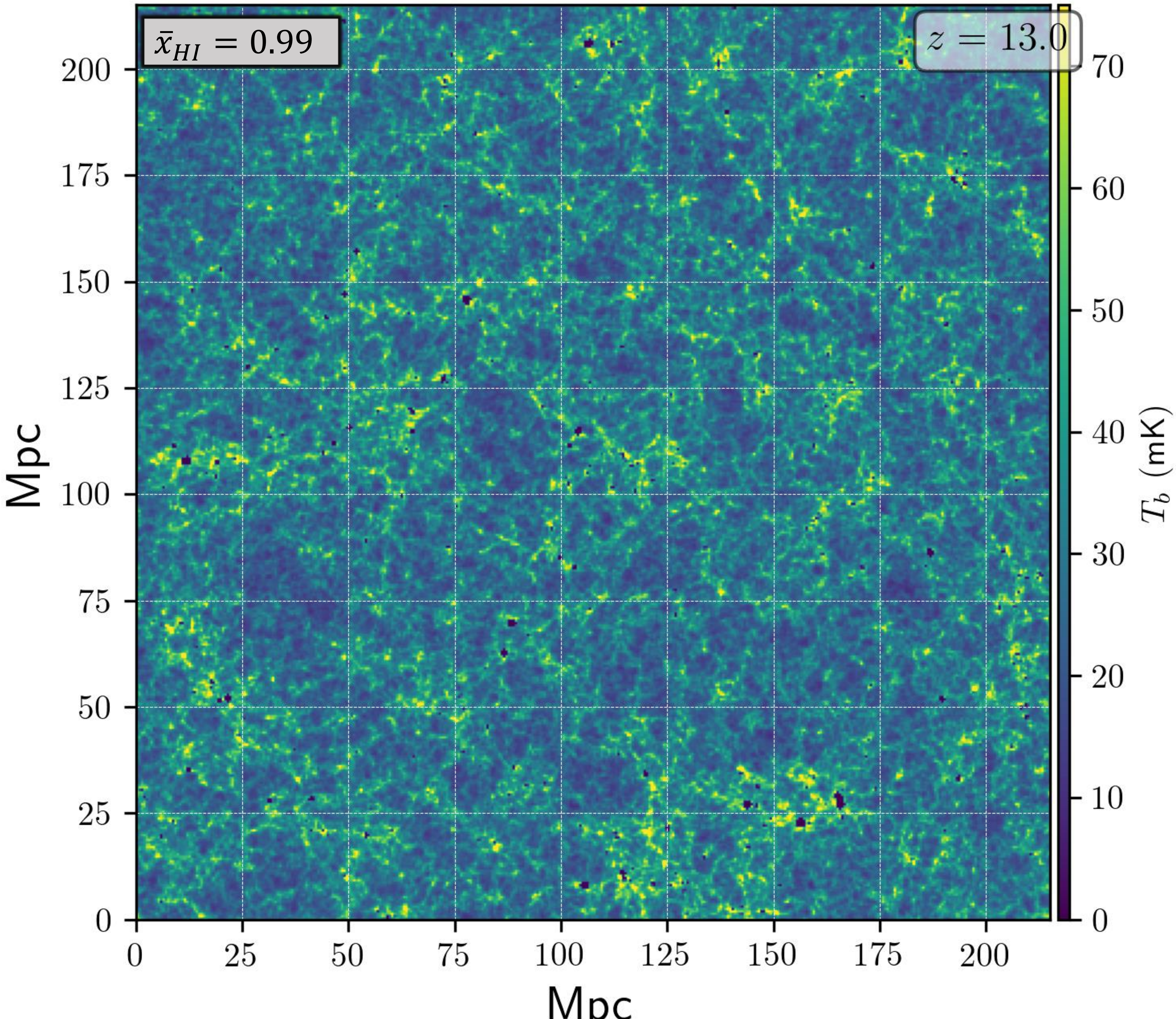
45 realizations

Simulating EoR (ReionYuga)

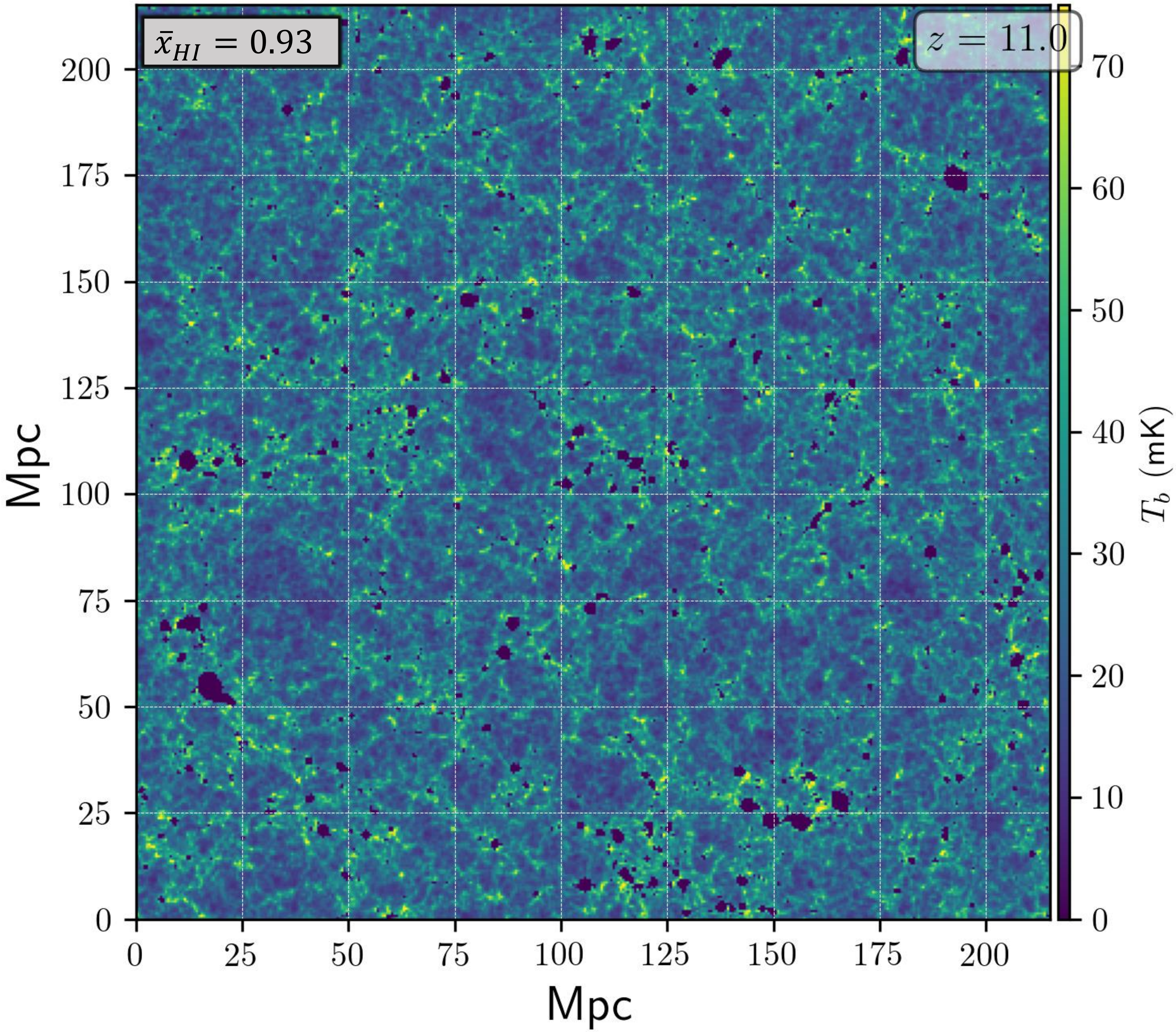
- Cube vol $= [215\ Mpc]^3$ = $[384$ grids$]^3$
- $M_{min} = 1.09 \times 10^9\ M_\odot$
- $N_{ion} = 23.21$
- $R_{mfp} = 20\ Mpc$

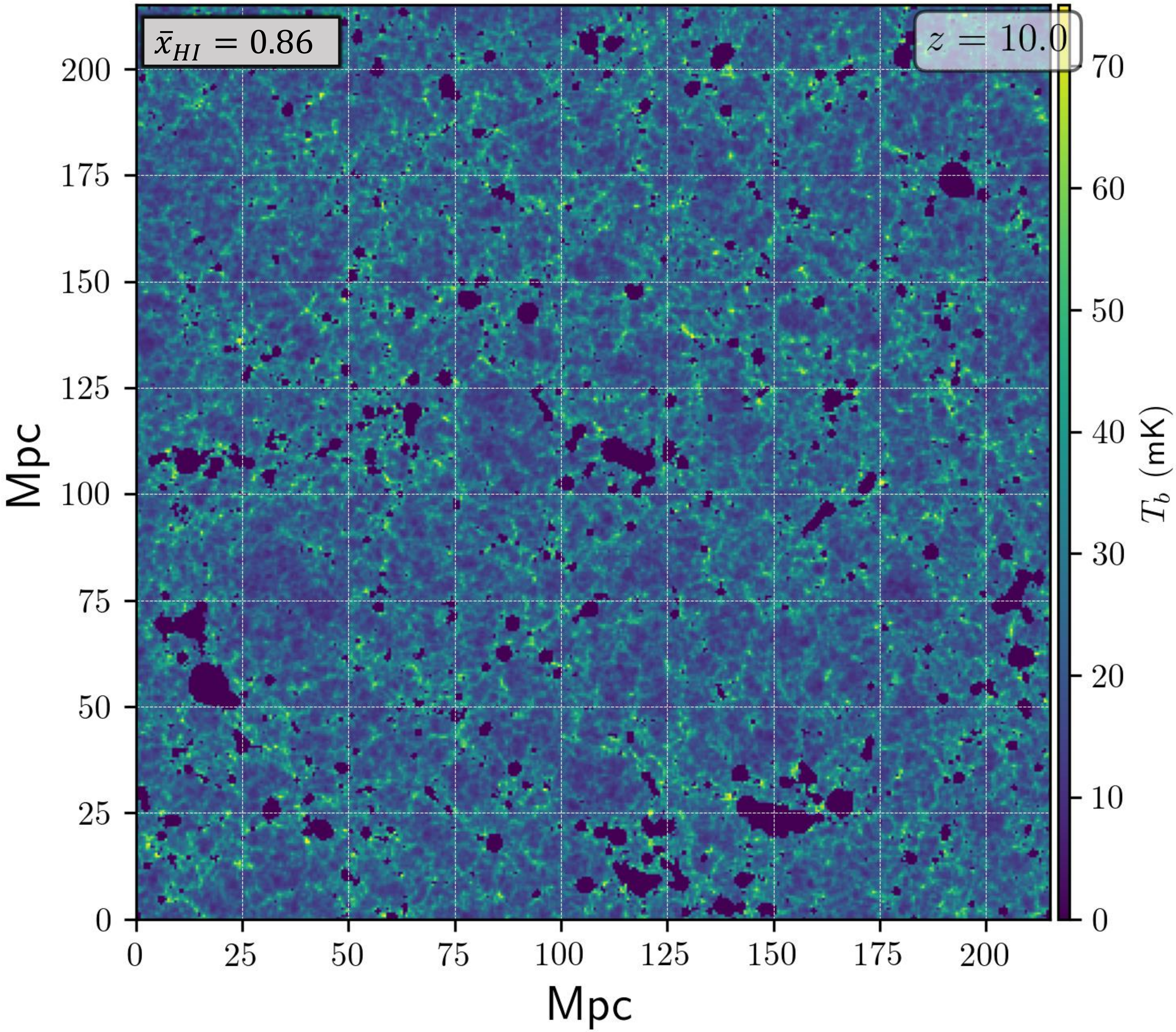
- EoR redshifts = 13 to 6
- $\bar{x}_{HI} = 0.5\ at\ z = 8$

45 realizations

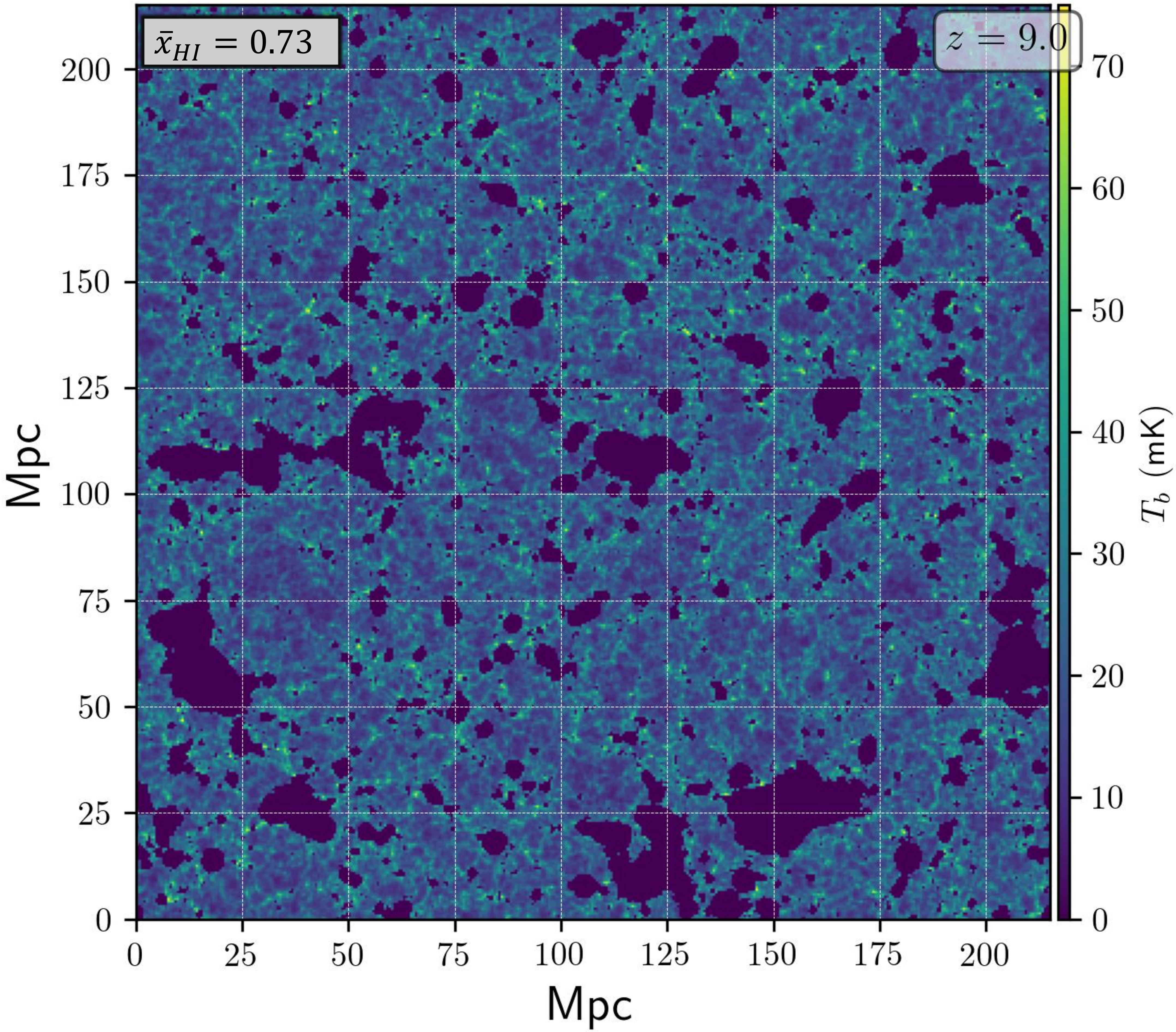


$\bar{x}_{HI} = 0.99$
$z = 13.0$
Mpc
Mpc
T_b (mK)
0
25
50
75
100
125
150
175
200
0
10
20
30
40
50
60
70

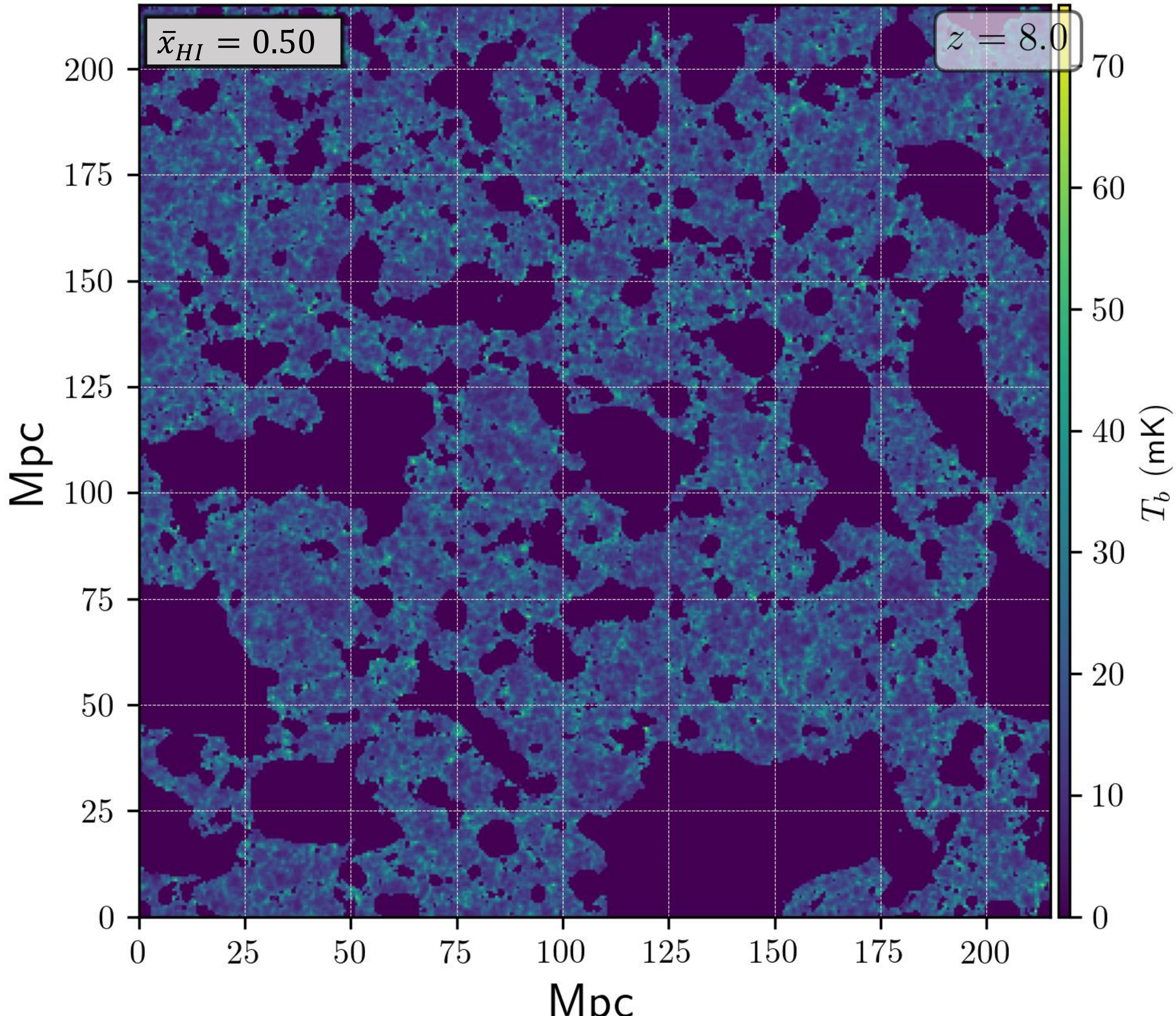

$\bar{x}_{HI} = 0.93$
$z = 11.0$
200
175
150
125
100
75
50
25
0
Mpc
0
25
50
75
100
125
150
175
200
Mpc
70
60
50
40
30
20
10
0
T_b (mK)



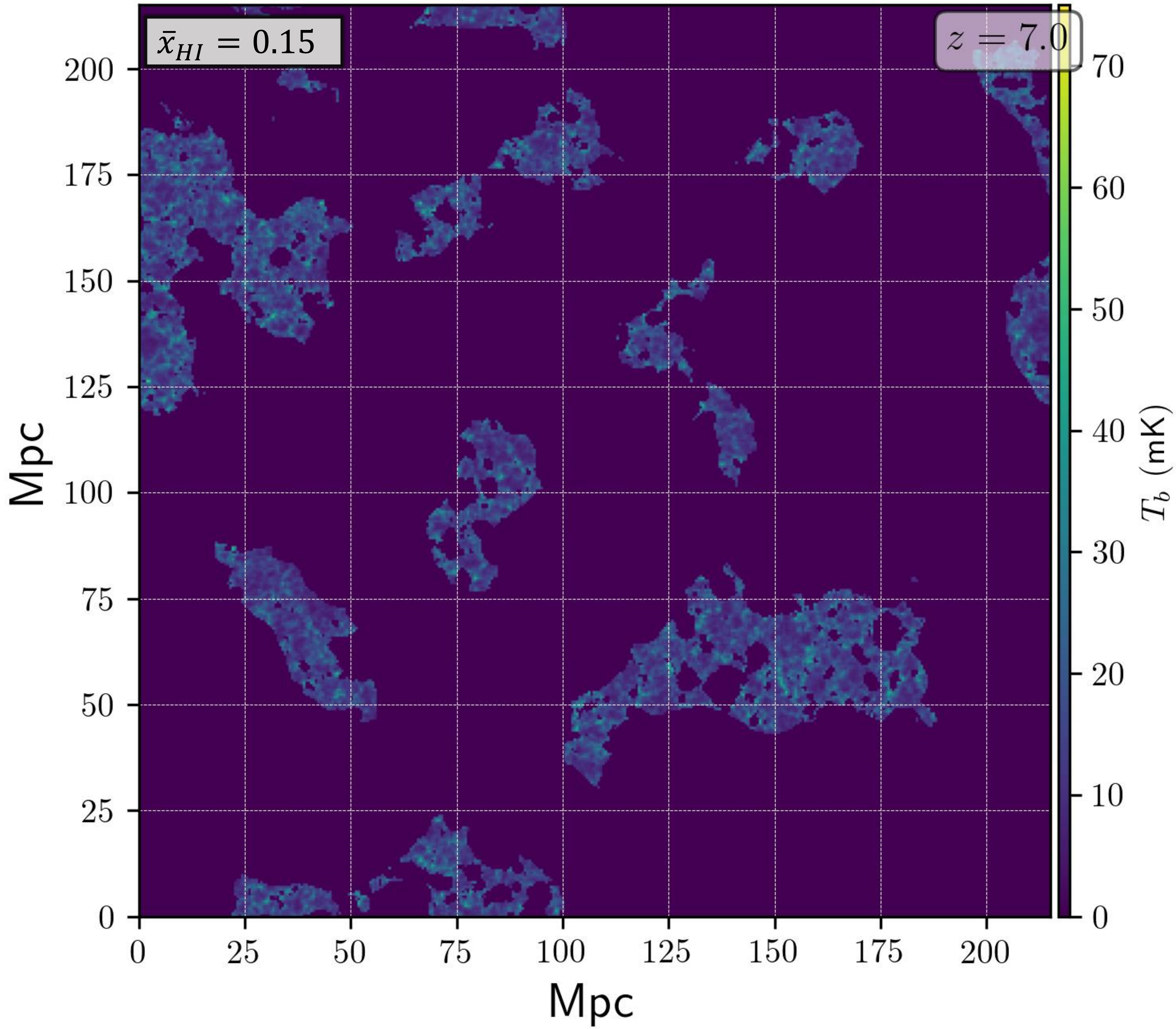
$\bar{x}_{HI} = 0.86$
$z = 10.0$
Mpc
Mpc
T_b (mK)
0
25
50
75
100
125
150
175
200
0
10
20
30
40
50
60
70



$\bar{x}_{HI} = 0.73$
$z = 9.0$
Mpc
0
25
50
75
100
125
150
175
200
Mpc
T_b (mK)
0
10
20
30
40
50
60
70



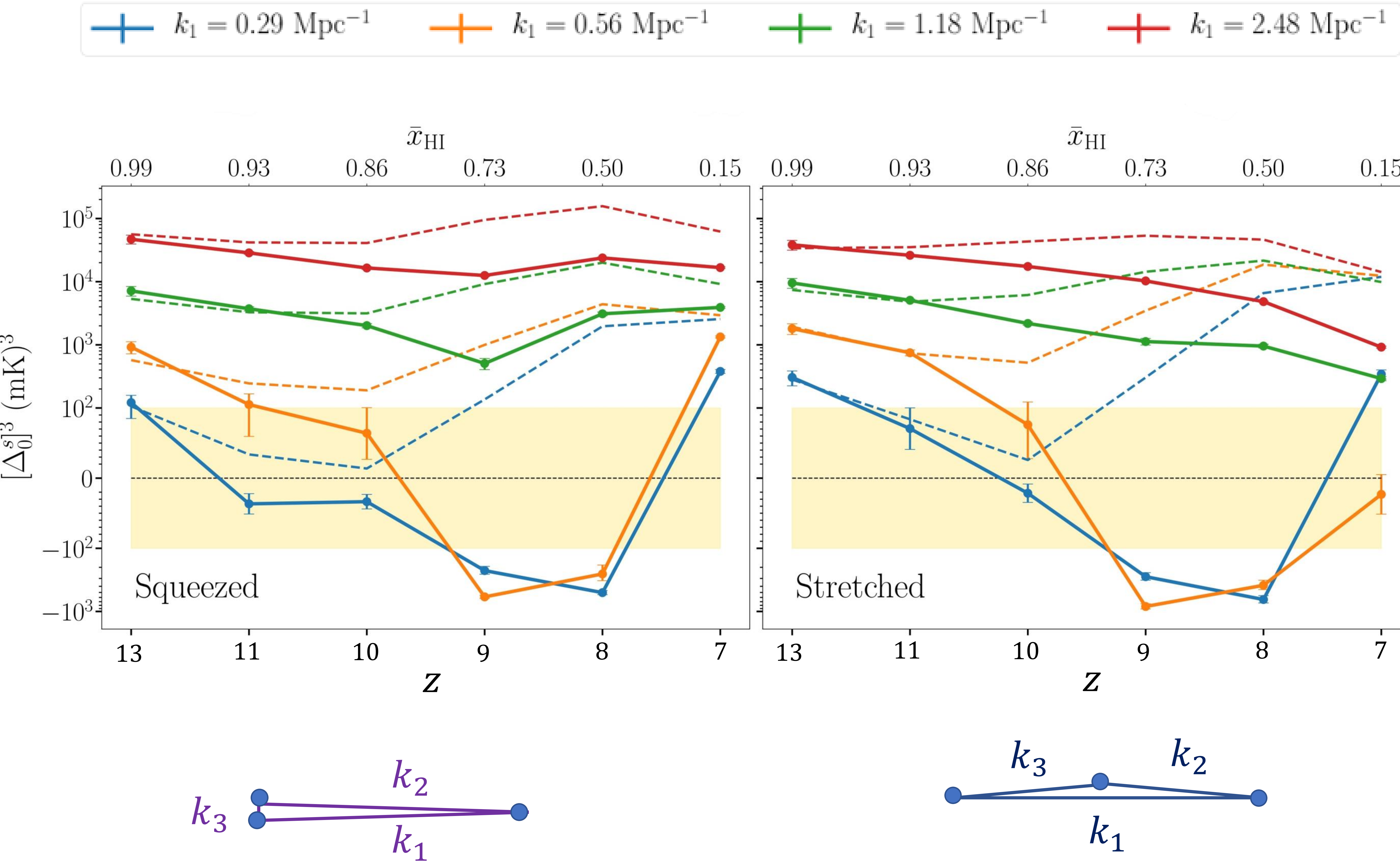
$\bar{x}_{HI} = 0.50$
$z = 8.0$
200
175
150
125
100
75
50
25
0
Mpc
0
25
50
75
100
125
150
175
200
Mpc
70
60
50
40
30
20
10
0
T_b (mK)



$\bar{x}_{HI} = 0.15$
$z = 7.0$
Mpc
Mpc
T_b (mK)
0
25
50
75
100
125
150
175
200
0
10
20
30
40
50
60
70

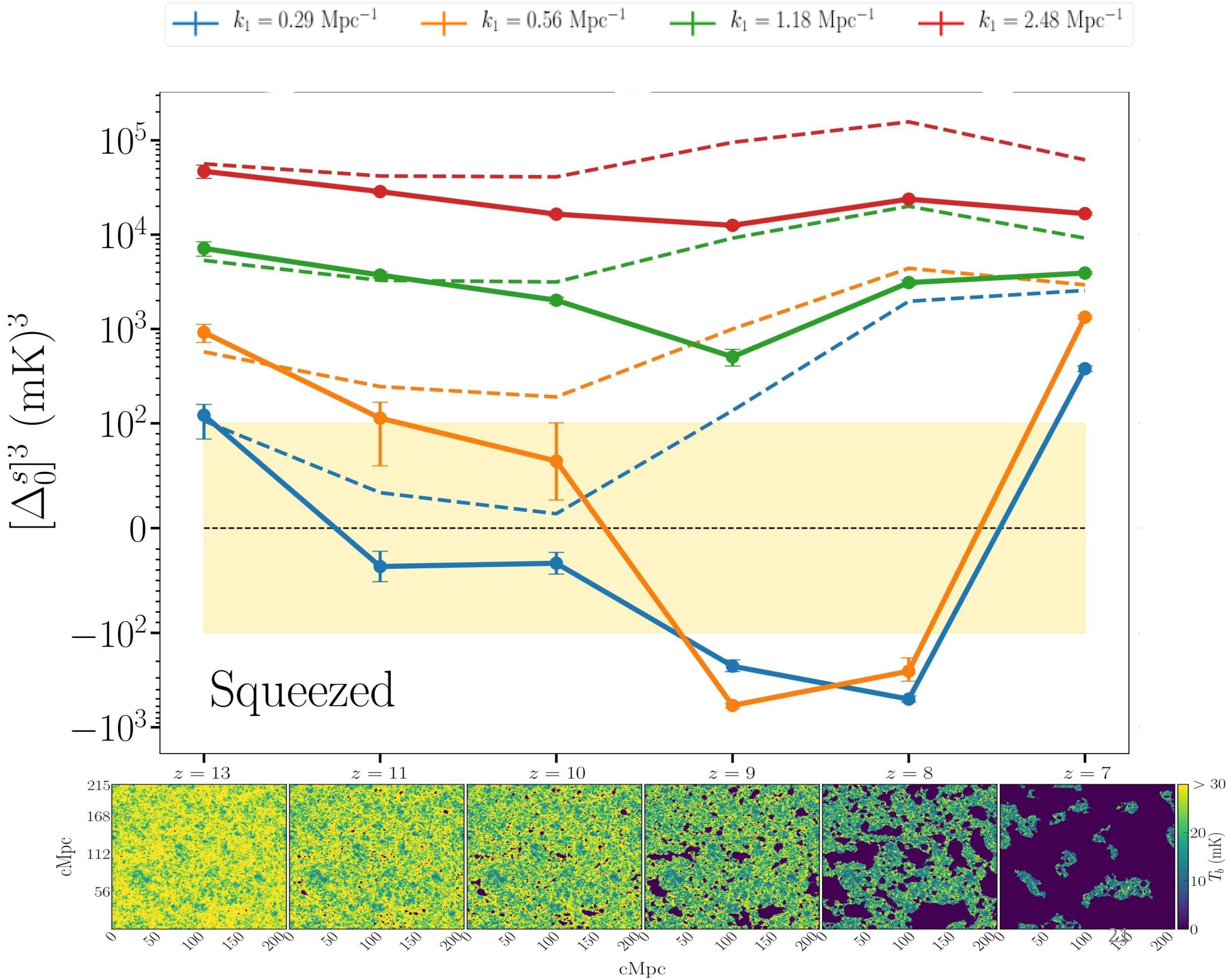
Redshift space bispectrum monopole

$$[\Delta_0^s]^3 = \frac{k_1^6 B_0^0(k_1, k_2, k_3)}{(2\pi^2)^2}$$

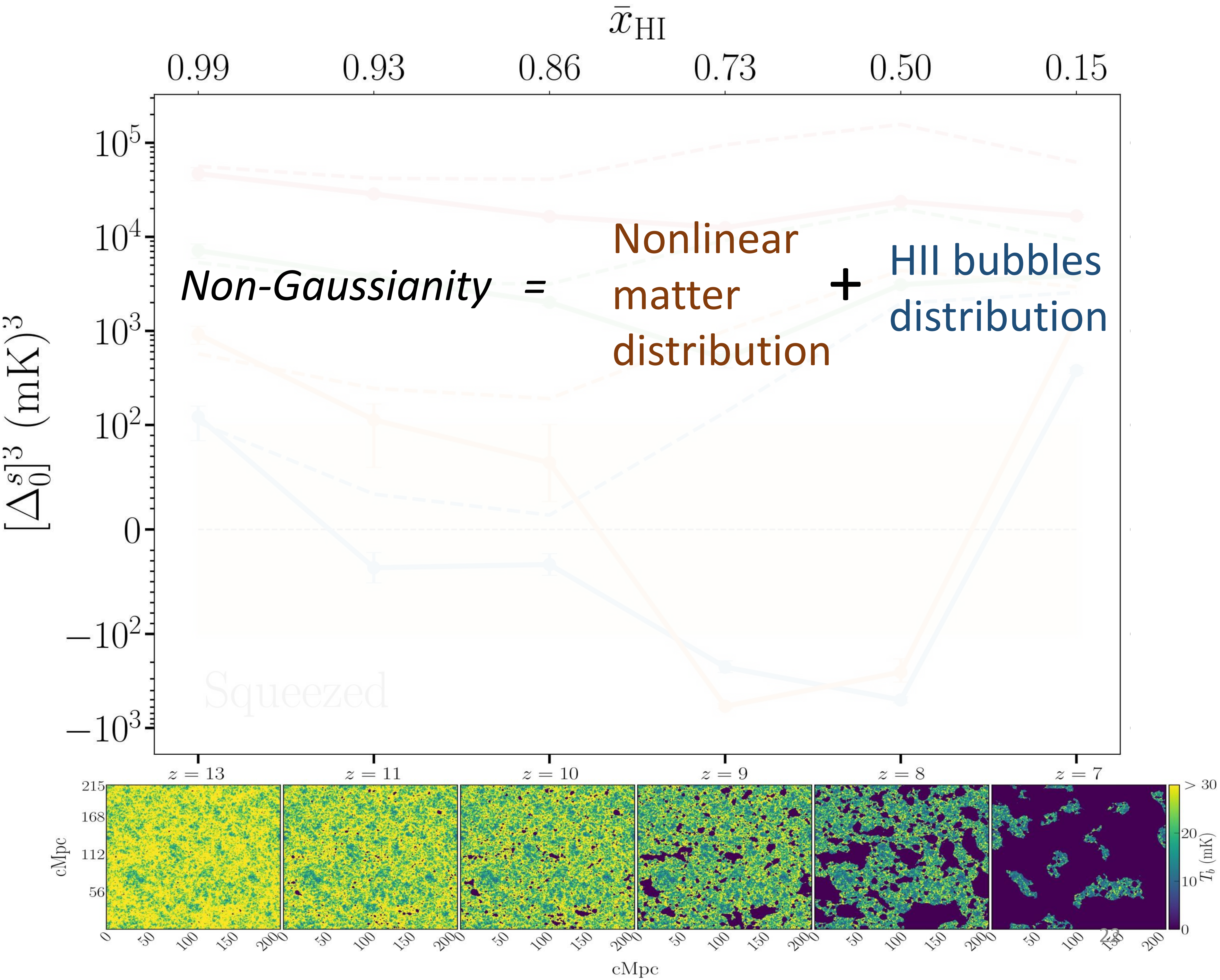


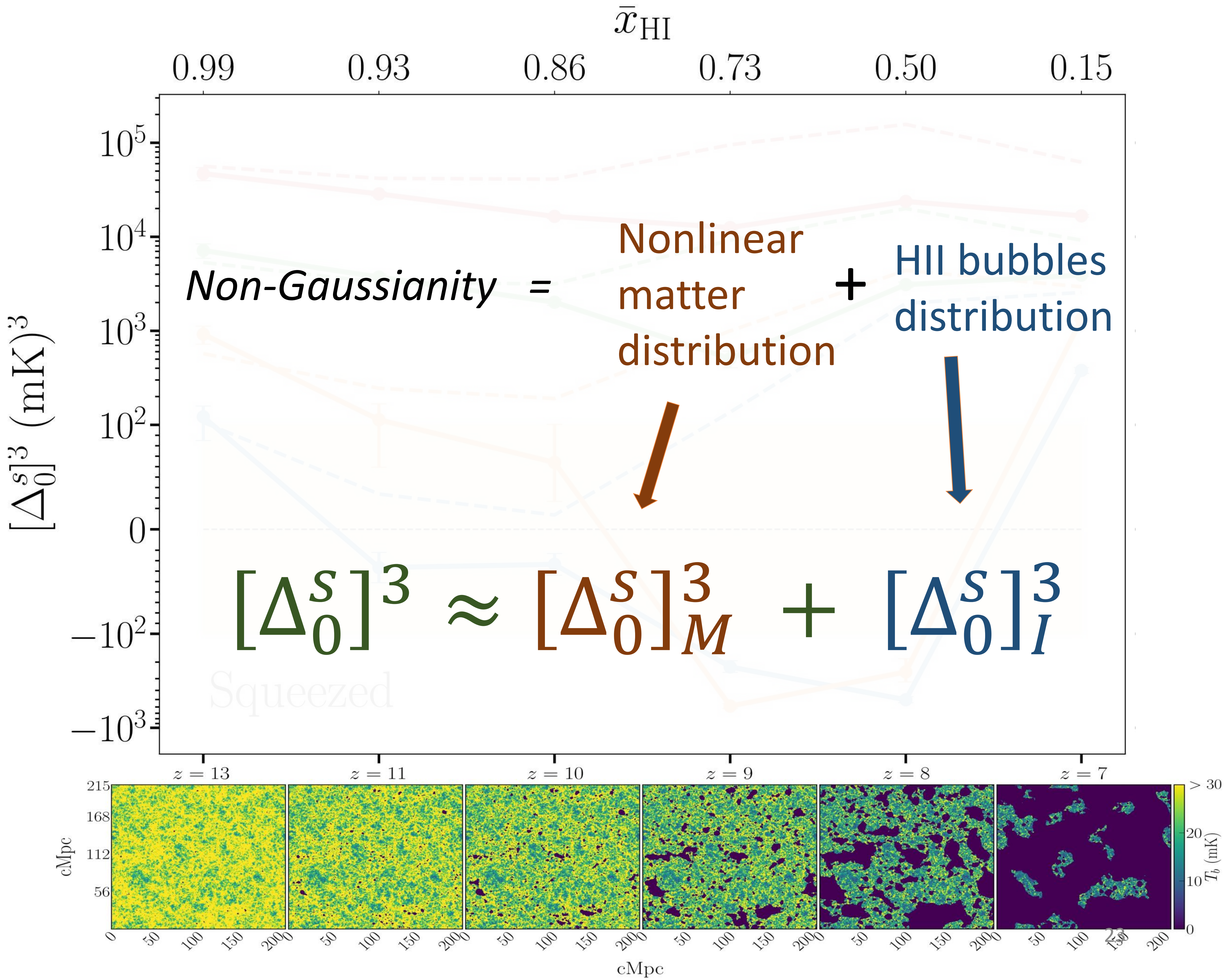
$(k_1 \approx k_2\ \&\ k_3 \approx 0)$

$(k_1 \approx 2k_2\ \&\ k_3 \approx k_2)$

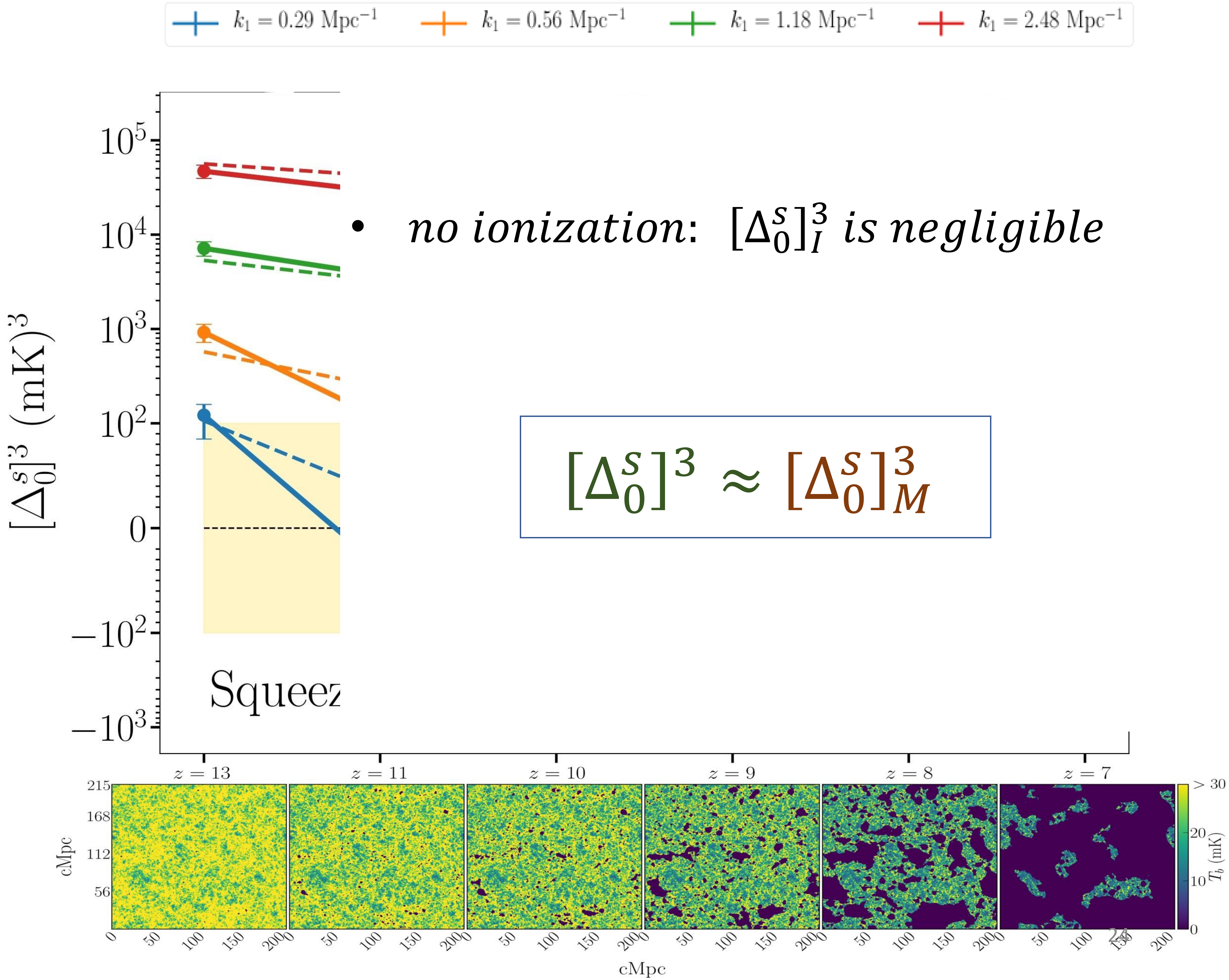

$k_1 = 0.29$ Mpc^{-1}
$k_1 = 0.56$ Mpc^{-1}
$k_1 = 1.18$ Mpc^{-1}
$k_1 = 2.48$ Mpc^{-1}
$[\Delta_0^s]^3$ (mK)^3
10^5
10^4
10^3
10^2
0
-10^2
-10^3
Squeezed
$z = 13$
$z = 11$
$z = 10$
$z = 9$
$z = 8$
$z = 7$
cMpc
215
168
112
56
0
50
100
150
200
> 30
20
10
0
T_b (mK)
cMpc

21


Non-Gaussianity = Nonlinear matter distribution + HII bubbles distribution
$\bar{x}_{\mathrm{HI}}$
0.99
0.93
0.86
0.73
0.50
0.15
$[\Delta_0^s]^3$ (mK)^3
Squeezed
$z = 13$
$z = 11$
$z = 10$
$z = 9$
$z = 8$
$z = 7$
cMpc
T_b (mK)



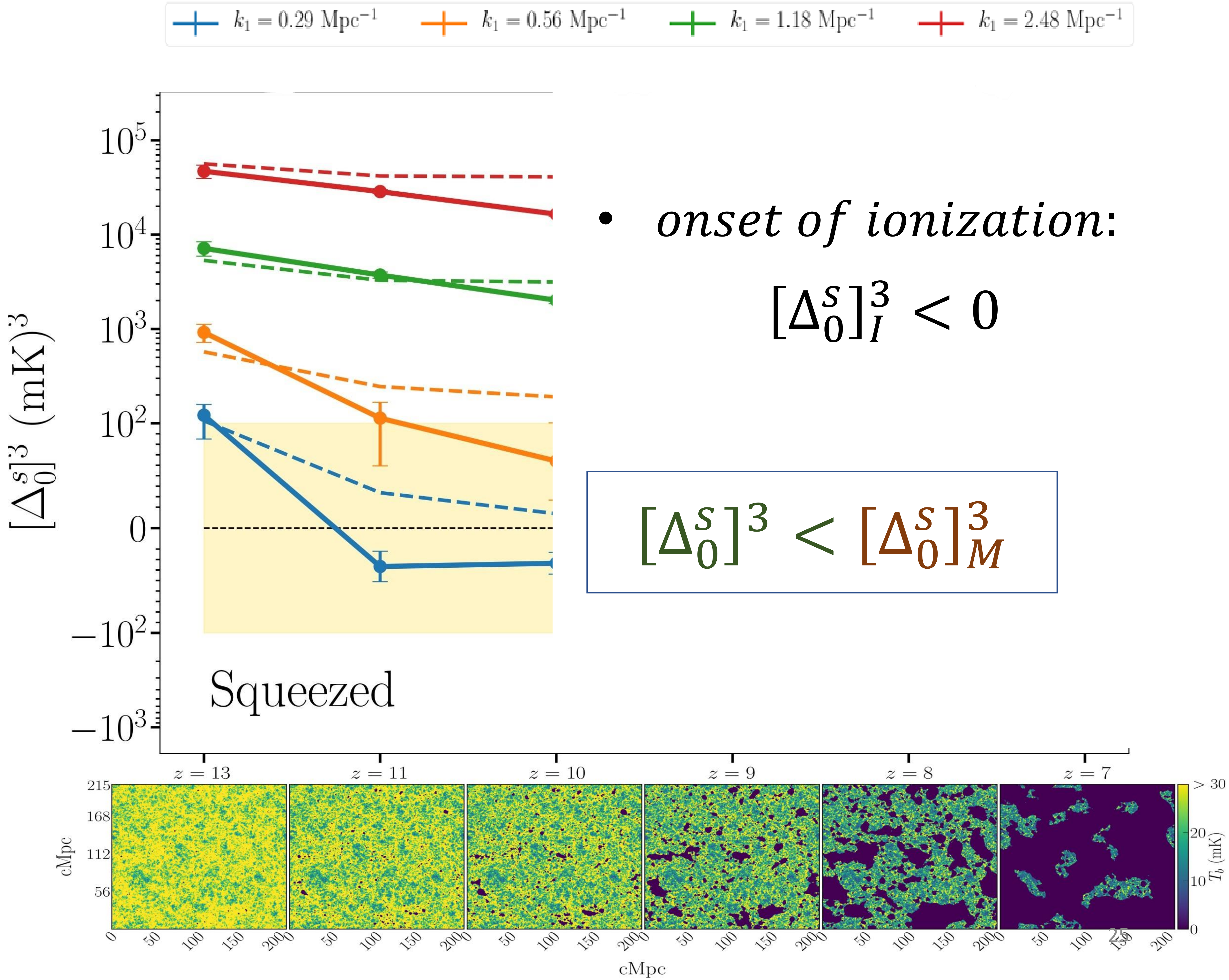
$\bar{x}_{\rm HI}$
0.99 0.93 0.86 0.73 0.50 0.15
$[\Delta_0^s]^3$ (mK)^3
Non-Gaussianity =
Nonlinear matter distribution
+
HII bubbles distribution
$[\Delta_0^s]^3 \approx [\Delta_0^s]^3_M + [\Delta_0^s]^3_I$
Squeezed
z = 13 z = 11 z = 10 z = 9 z = 8 z = 7
cMpc
T_b (mK)



- *no ionization*: $[\Delta_0^s]_I^3$ *is negligible*

$$[\Delta_0^s]^3 \approx [\Delta_0^s]_M^3$$

24

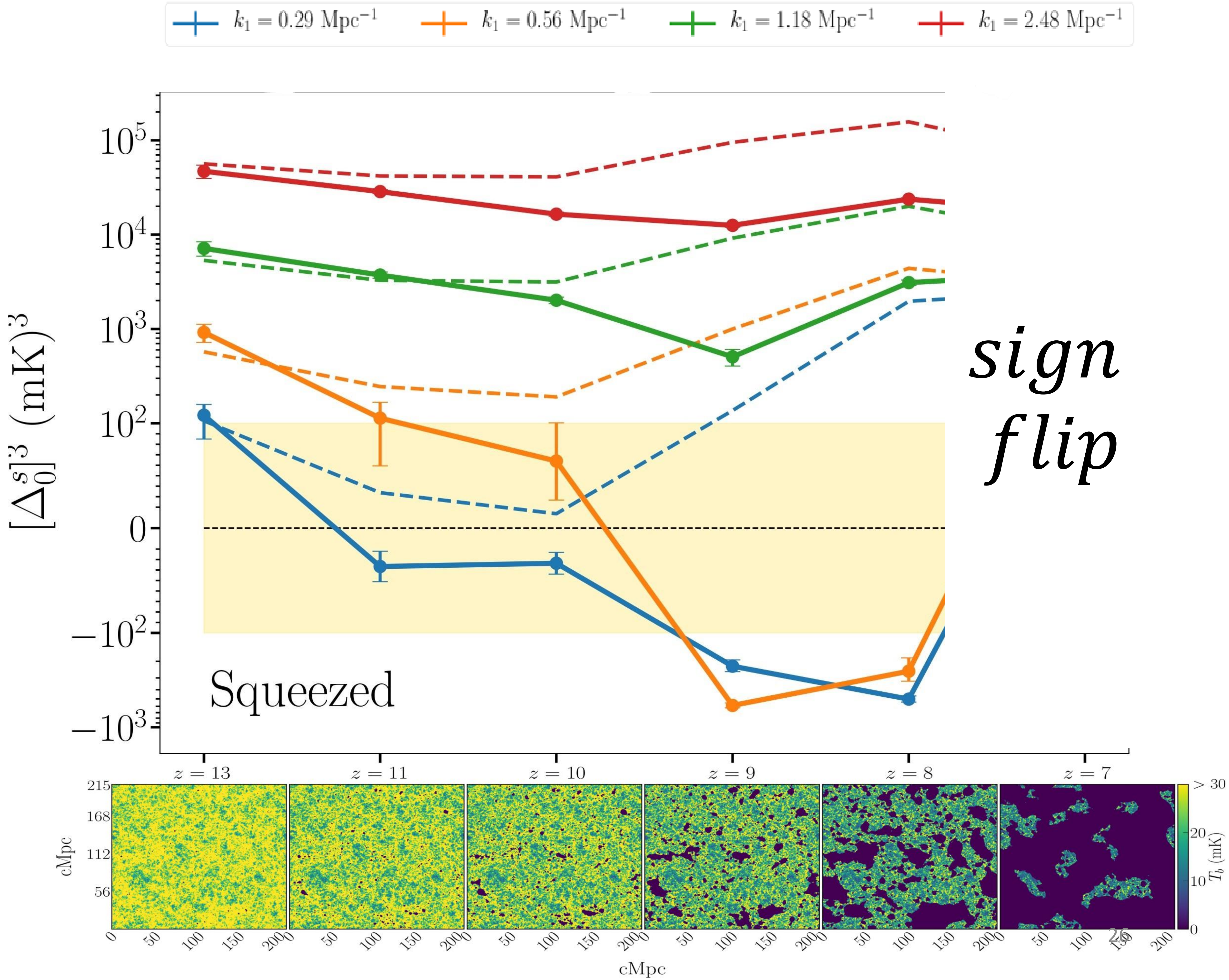


- *onset of ionization*:

$$[\Delta_0^s]_I^3 < 0$$

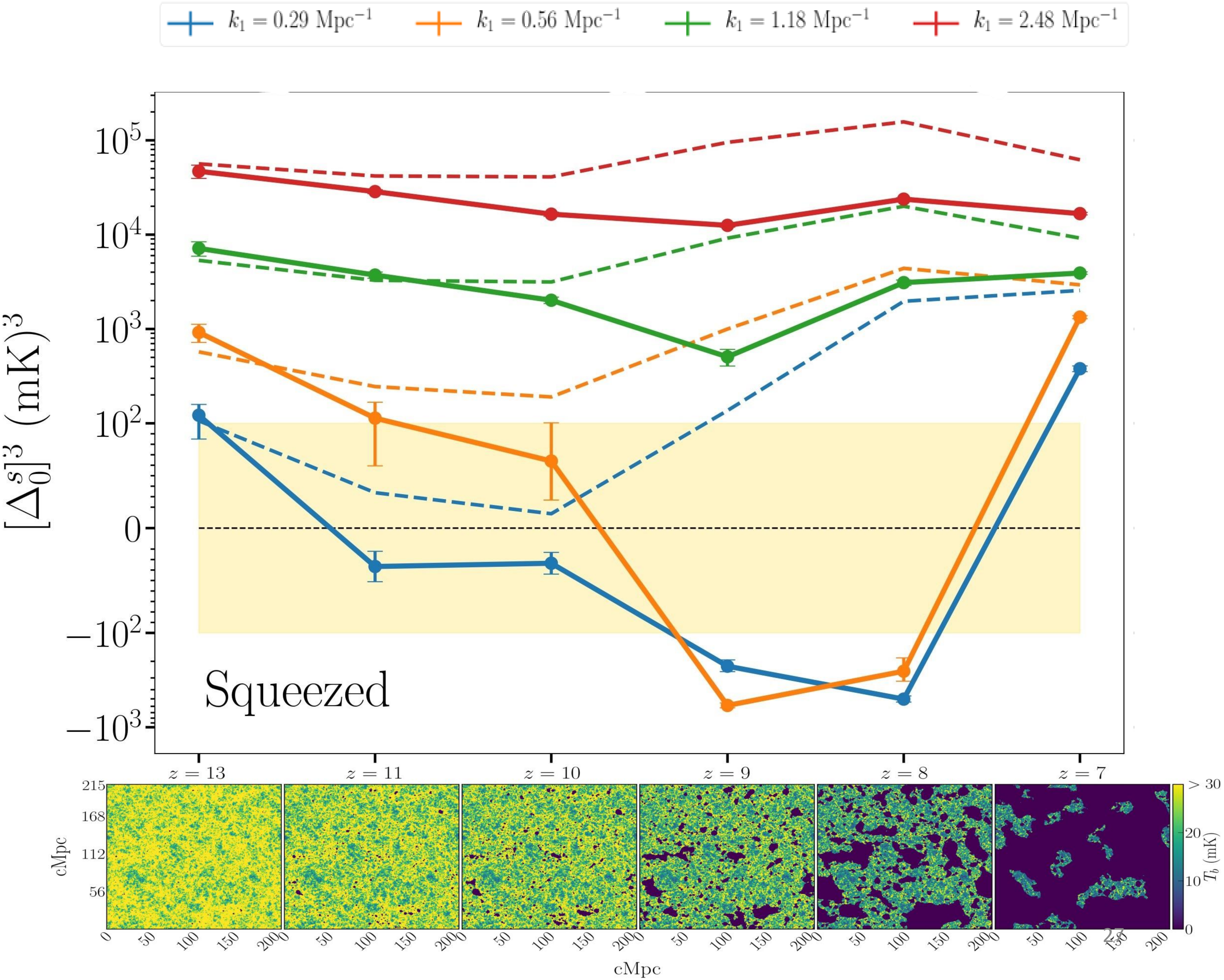
$$[\Delta_0^s]^3 < [\Delta_0^s]_M^3$$

25



$k_1 = 0.29$ Mpc^{-1}
$k_1 = 0.56$ Mpc^{-1}
$k_1 = 1.18$ Mpc^{-1}
$k_1 = 2.48$ Mpc^{-1}
$[\Delta_0^s]^3$ (mK)^3
Squeezed
$z = 13$
$z = 11$
$z = 10$
$z = 9$
$z = 8$
$z = 7$
cMpc
T_b (mK)

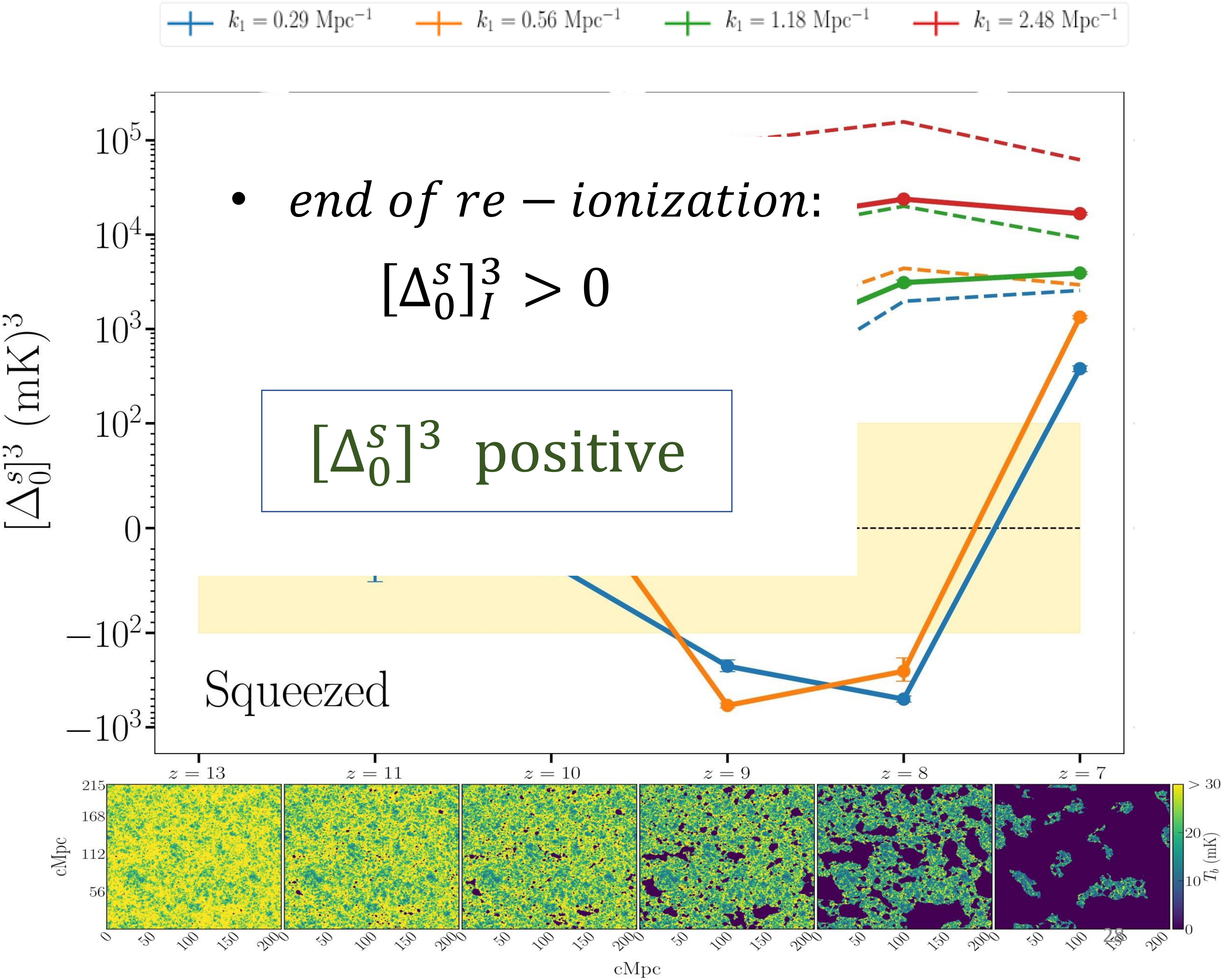
sign flip



$k_1 = 0.29$ Mpc^{-1}
$k_1 = 0.56$ Mpc^{-1}
$k_1 = 1.18$ Mpc^{-1}
$k_1 = 2.48$ Mpc^{-1}
$[\Delta_0^s]^3$ (mK)^3
10^5
10^4
10^3
10^2
0
-10^2
-10^3
Squeezed
$z = 13$
$z = 11$
$z = 10$
$z = 9$
$z = 8$
$z = 7$
215
168
112
56
cMpc
0
50
100
150
200
cMpc
> 30
20
10
0
T_b (mK)

- *end of re − ionization*: $[\Delta_0^s]_I^3 > 0$

$[\Delta_0^s]^3$ positive



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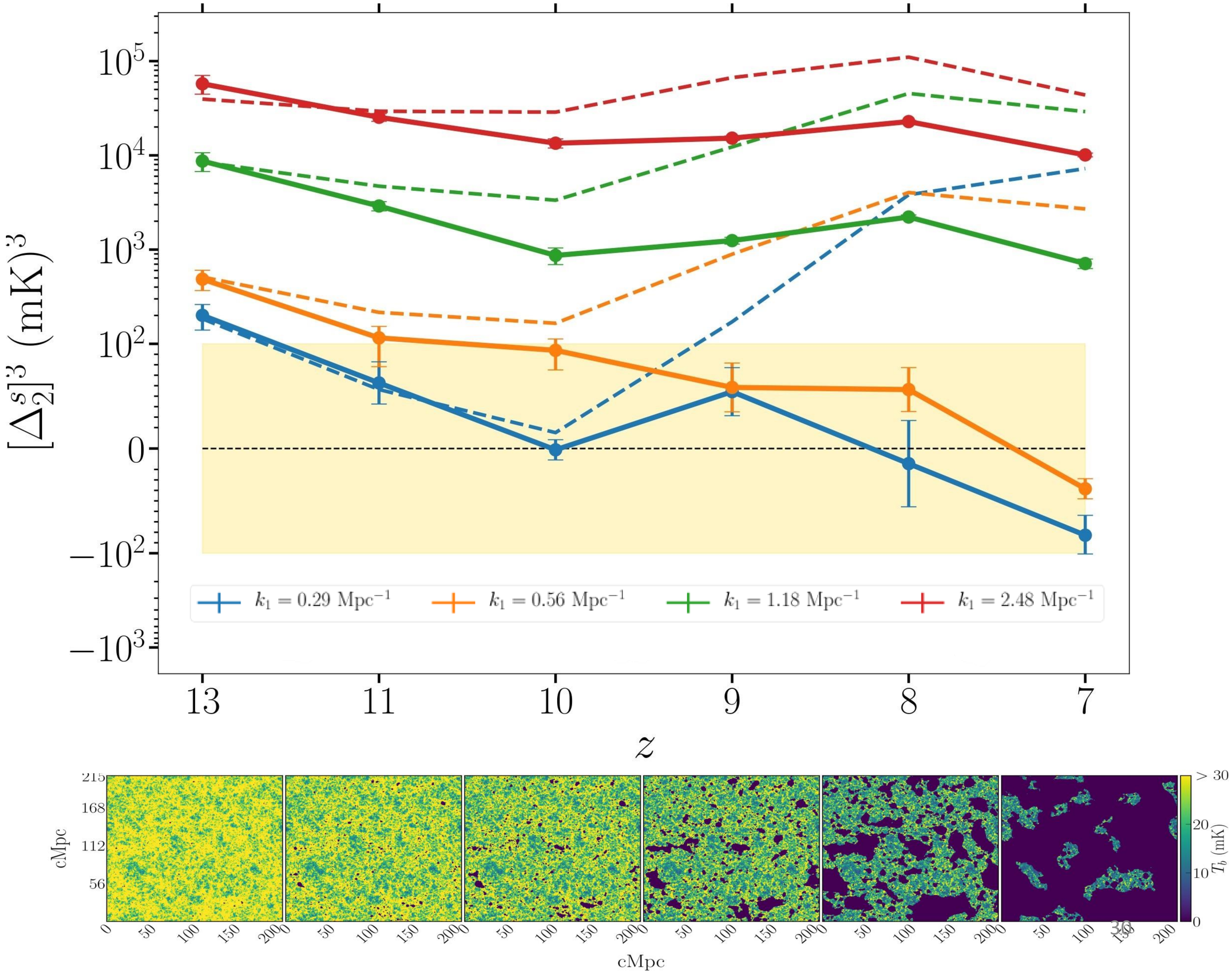
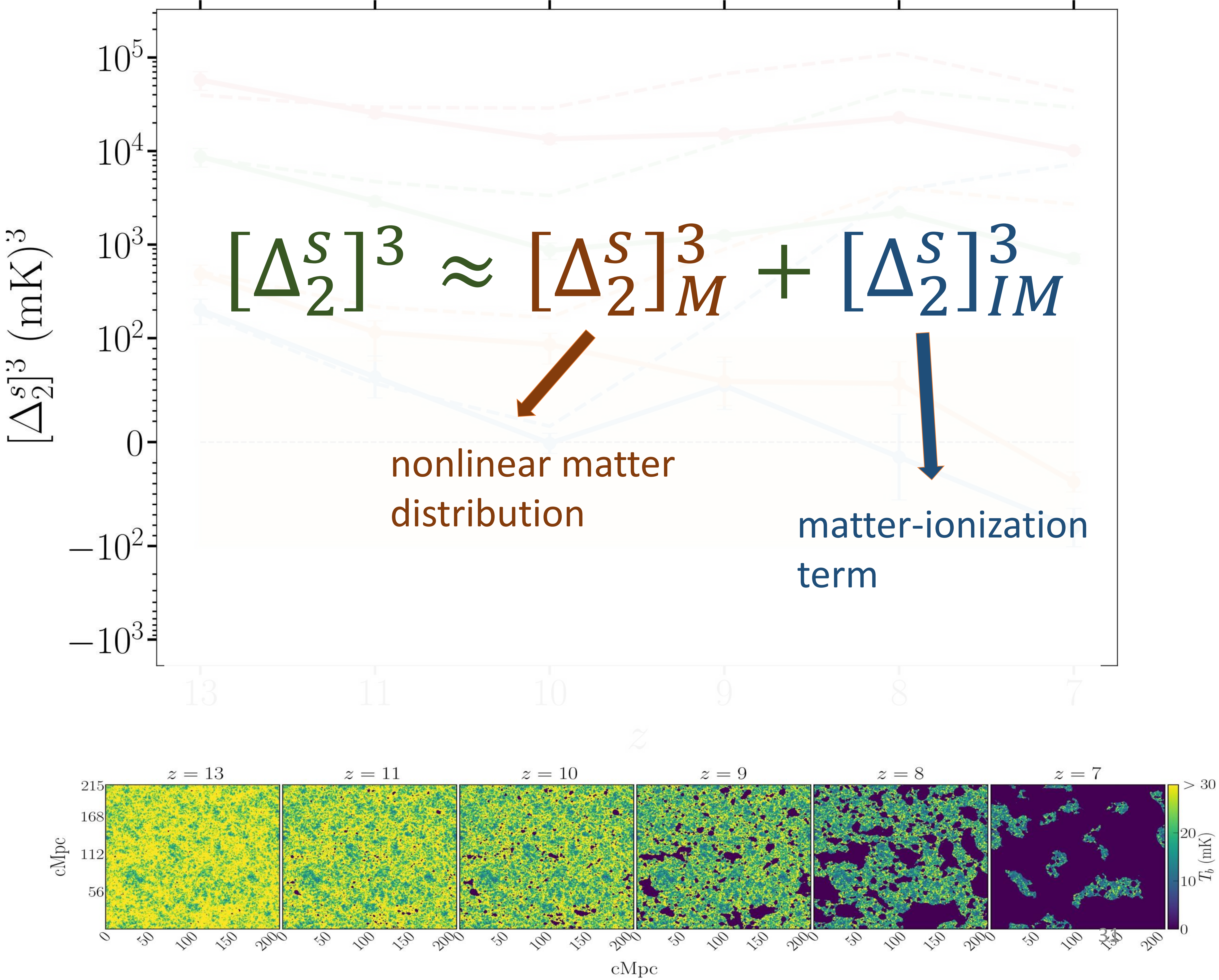
Redshift space bispectrum Quadrupole

$$[\Delta_2^s]^3 = \frac{k_1^6 B_2^0(k_1, k_2, k_3)}{(2\pi^2)^2}$$



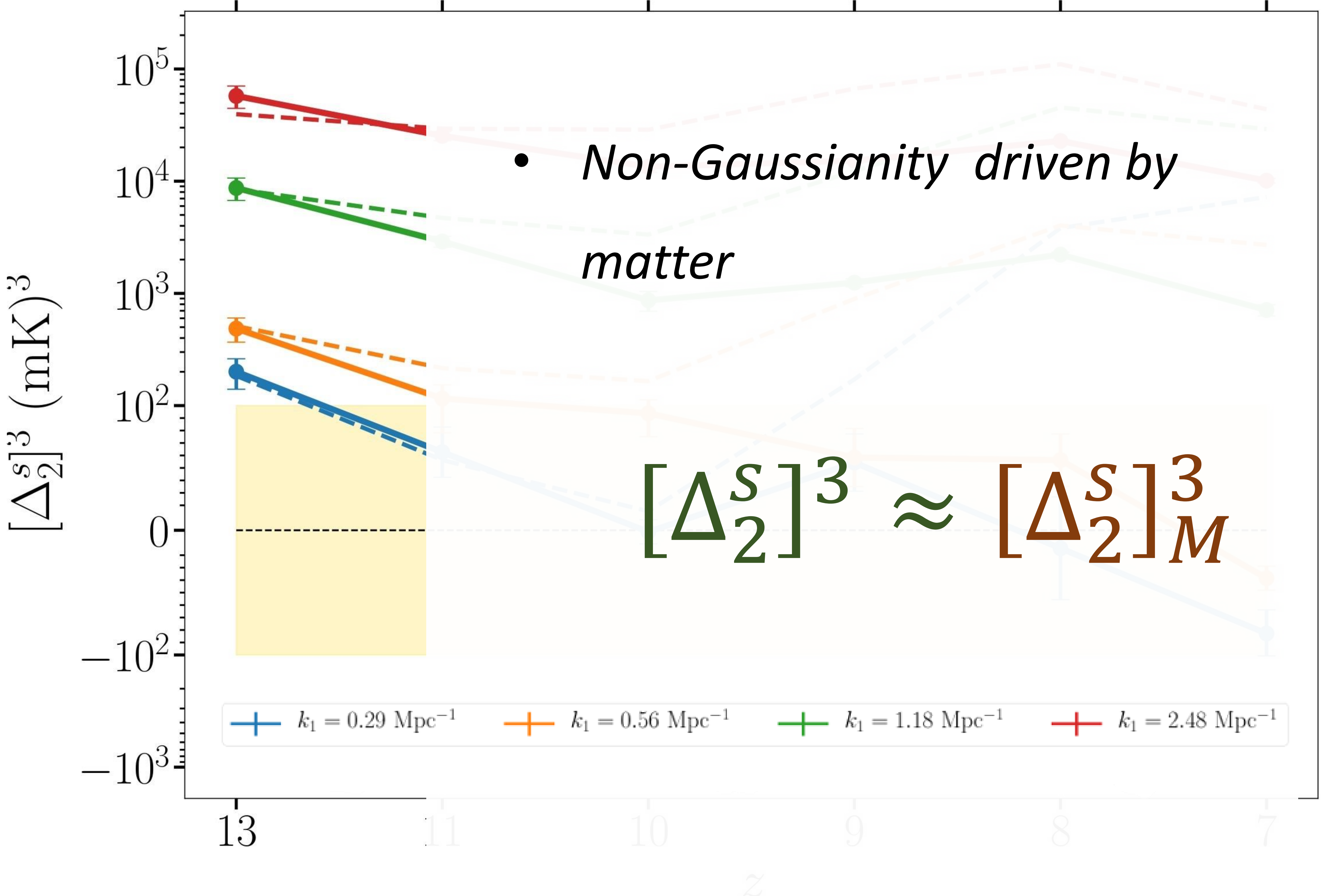
$[\Delta_2^s]^3\ (\mathrm{mK})^3$
10^5
10^4
10^3
10^2
0
-10^2
-10^3
13
11
10
9
8
7
z
$k_1 = 0.29\ \mathrm{Mpc}^{-1}$
$k_1 = 0.56\ \mathrm{Mpc}^{-1}$
$k_1 = 1.18\ \mathrm{Mpc}^{-1}$
$k_1 = 2.48\ \mathrm{Mpc}^{-1}$
cMpc
215
168
112
56
0
50
100
150
200
cMpc
T_b (mK)
> 30
20
10
0

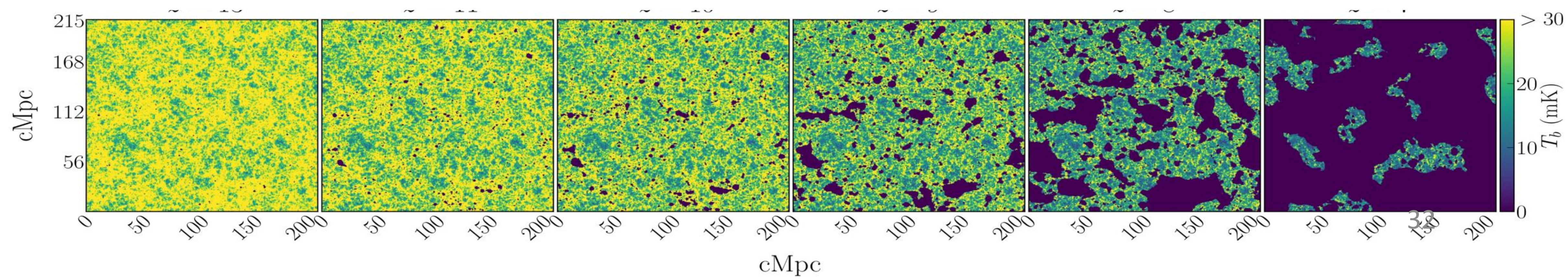


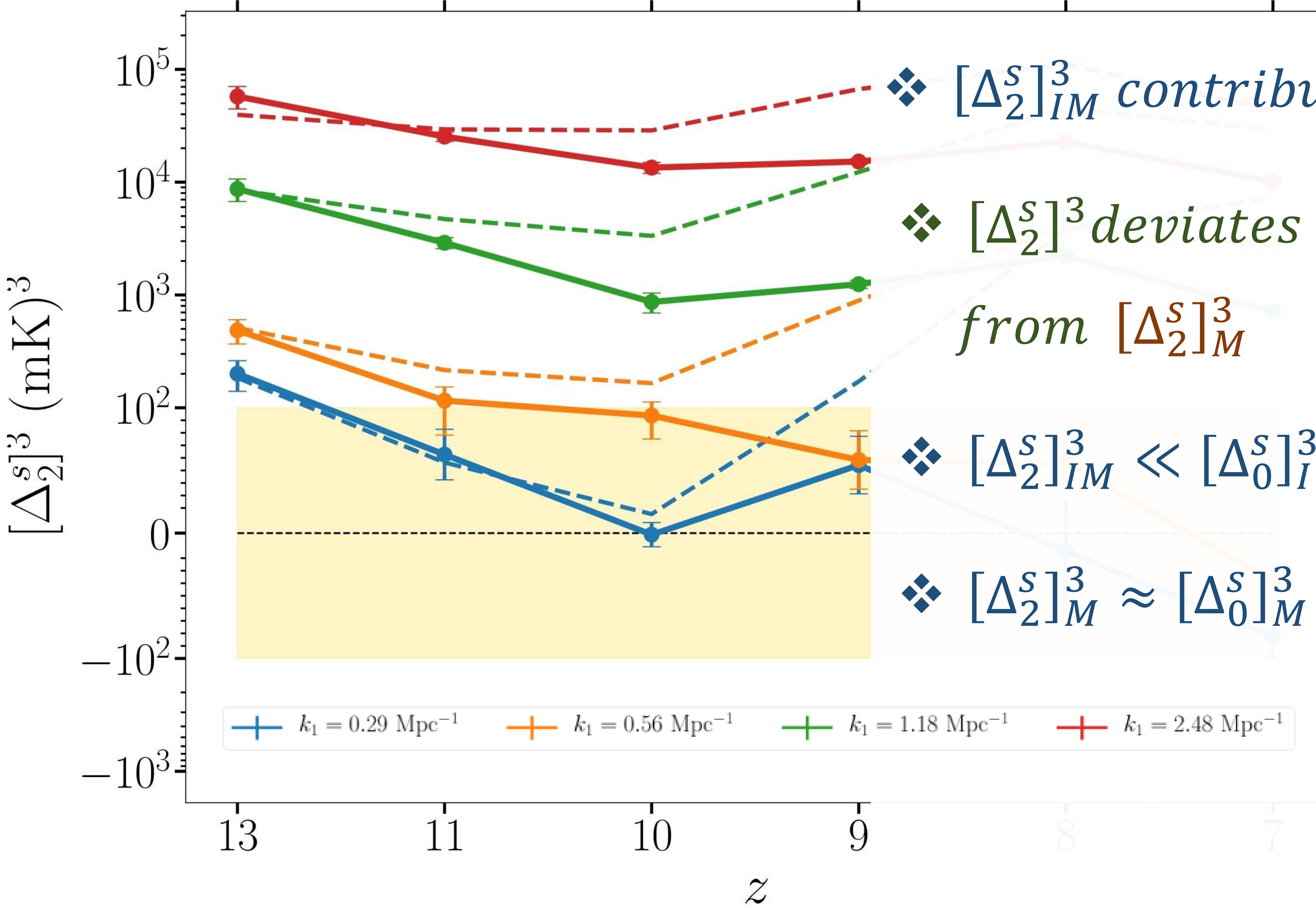
$[\Delta_2^s]^3 \approx [\Delta_2^s]^3_M + [\Delta_2^s]^3_{IM}$
nonlinear matter distribution
matter-ionization term
$[\Delta_2^s]^3$ (mK)^3
10^5
10^4
10^3
10^2
0
-10^2
-10^3
$z = 13$
$z = 11$
$z = 10$
$z = 9$
$z = 8$
$z = 7$
cMpc
215
168
112
56
0
50
100
150
200
cMpc
> 30
20
10
0
T_b (mK)

- *Non-Gaussianity driven by matter*

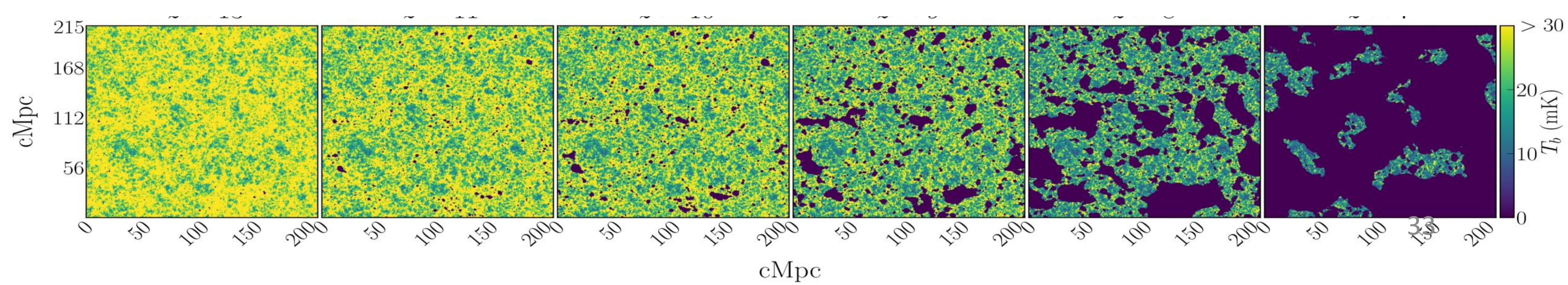
$$[\Delta_2^s]^3 \approx [\Delta_2^s]_M^3$$







- ❖ $[\Delta_2^s]^3_{IM}$ *contributes*
- ❖ $[\Delta_2^s]^3$ *deviates from* $[\Delta_2^s]^3_M$
- ❖ $[\Delta_2^s]^3_{IM} \ll [\Delta_0^s]^3_I$
- ❖ $[\Delta_2^s]^3_M \approx [\Delta_0^s]^3_M$



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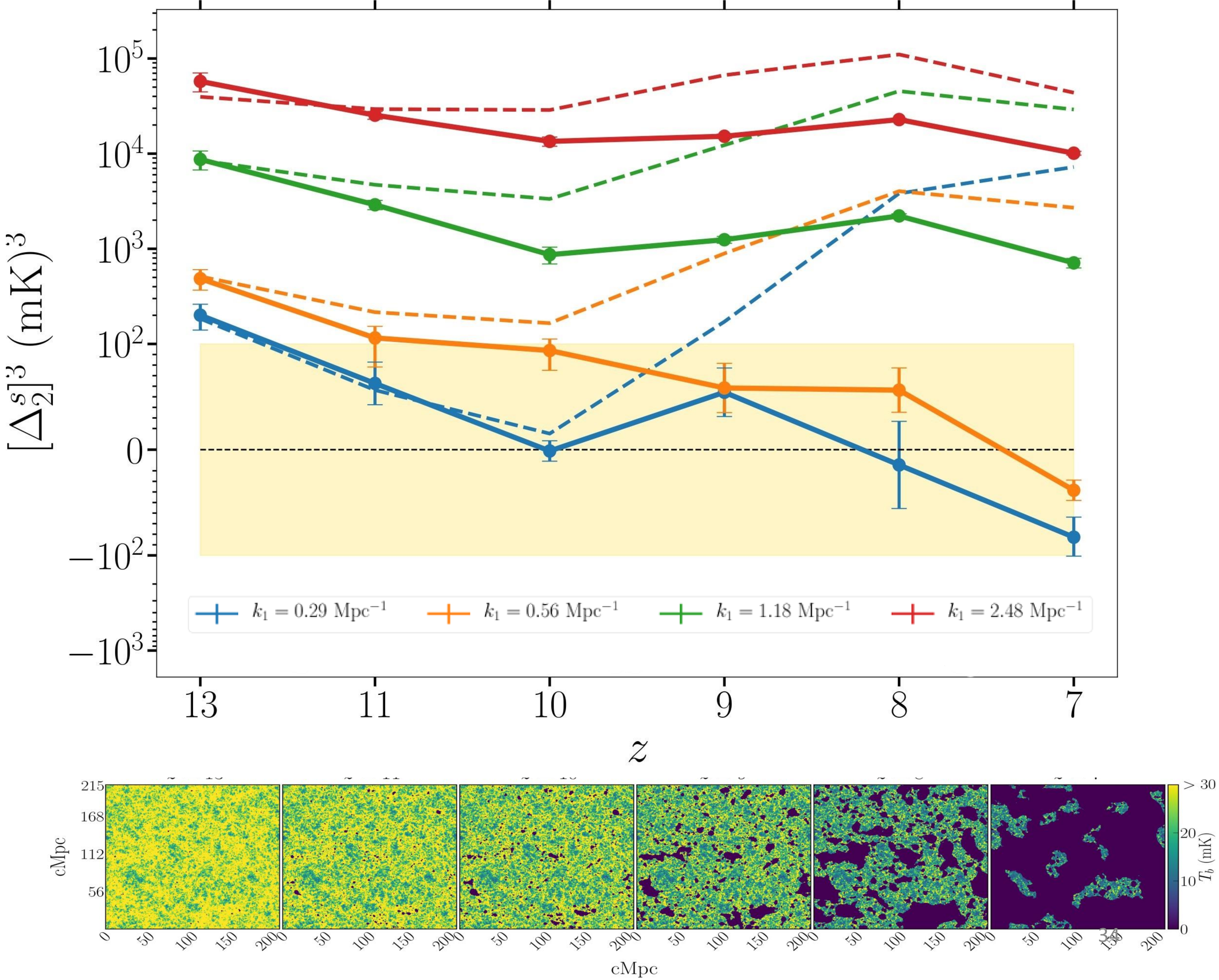
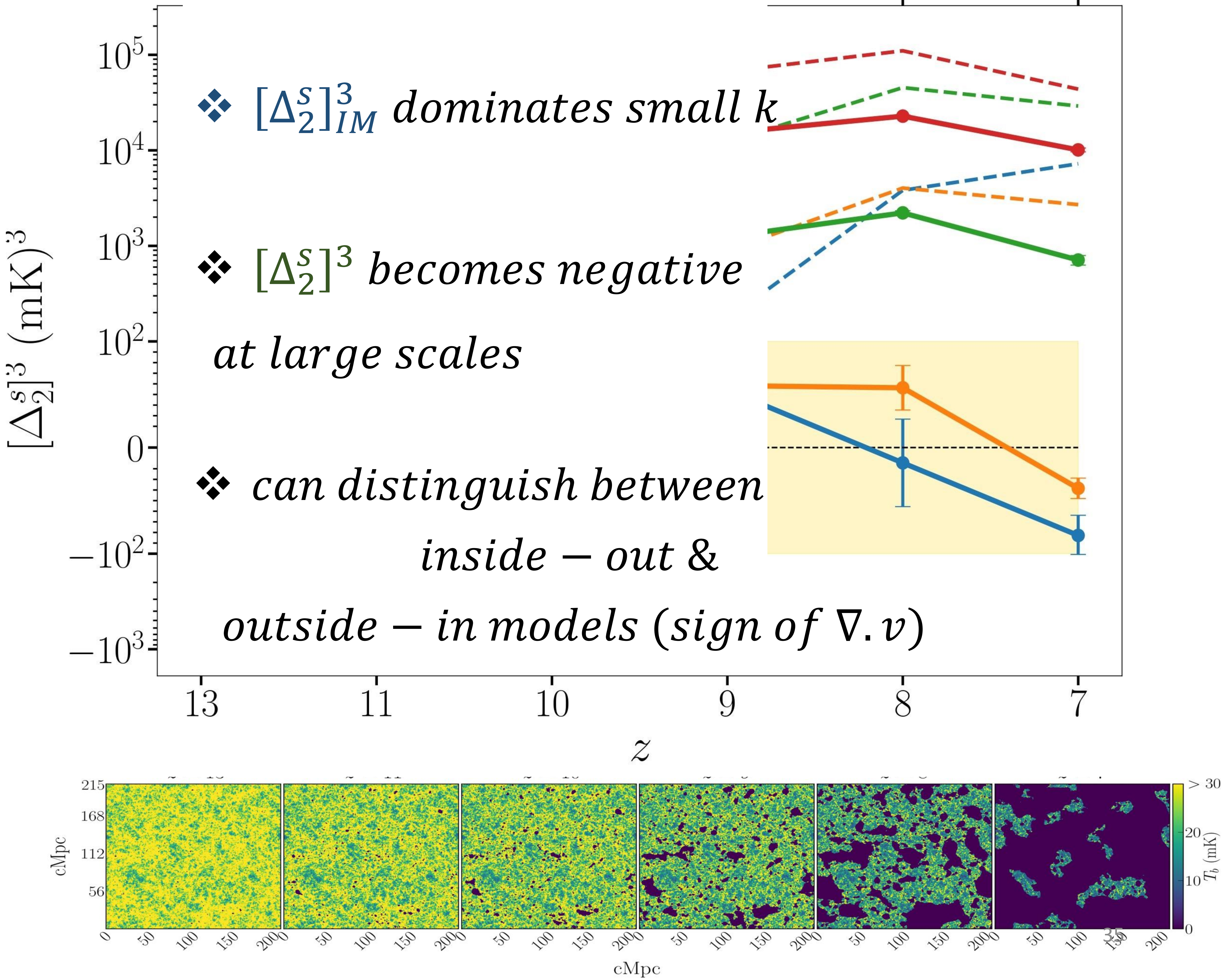

$[\Delta_2^s]^3$ (mK)^3
10^5
10^4
10^3
10^2
0
-10^2
-10^3
13
11
10
9
8
7
z
$k_1 = 0.29$ Mpc^{-1}
$k_1 = 0.56$ Mpc^{-1}
$k_1 = 1.18$ Mpc^{-1}
$k_1 = 2.48$ Mpc^{-1}
cMpc
cMpc
T_b (mK)
> 30

- ❖ $[\Delta_2^s]^3_{IM}$ *dominates small k*
- ❖ $[\Delta_2^s]^3$ *becomes negative at large scales*
- ❖ *can distinguish between inside − out & outside − in models (sign of* $\nabla . v$*)*

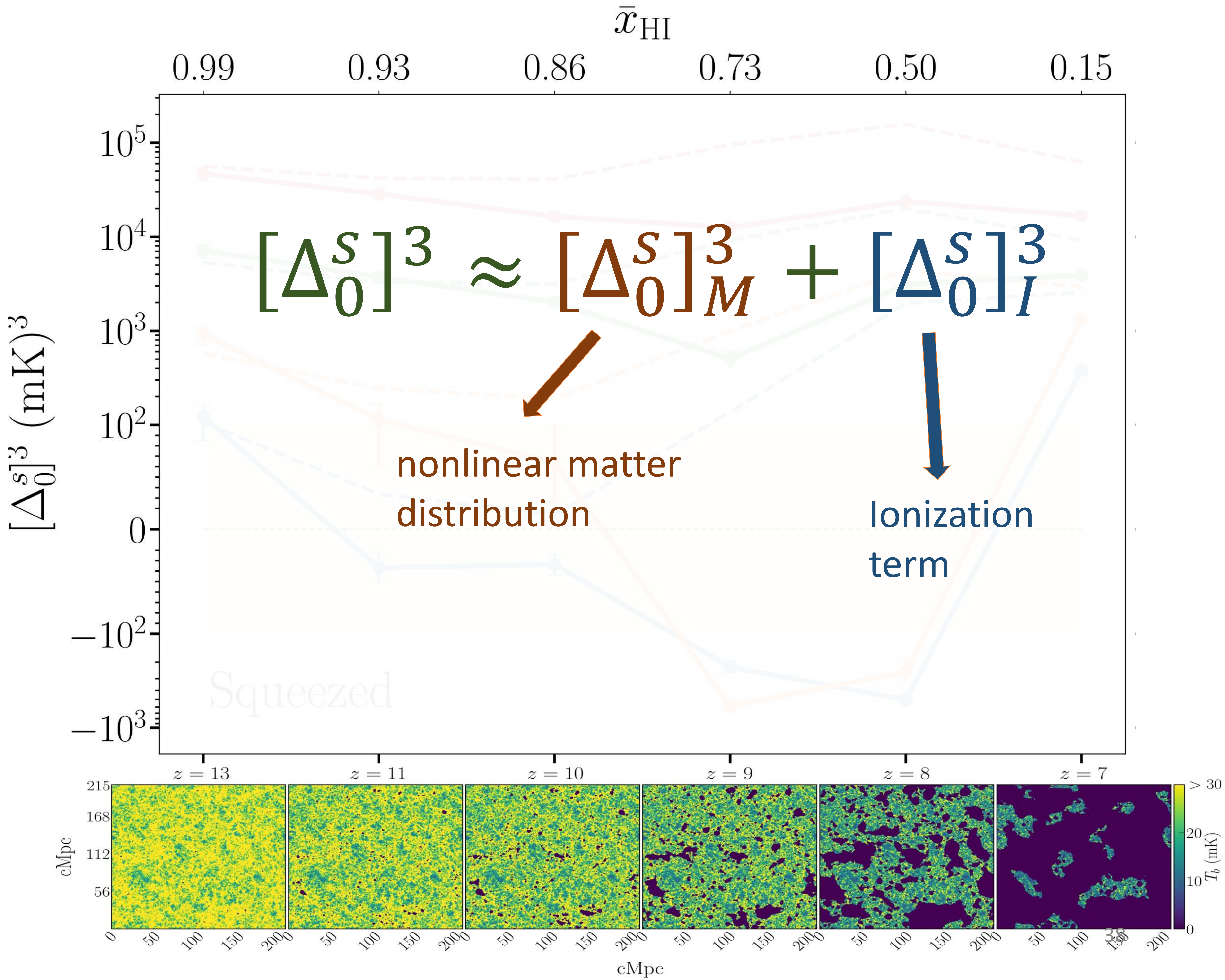


Summary

- Two sign flips of bispectrum monopole phase transitions of the universe
- Magnitude of quadrupole is similar to monopole
- Quadrupole moment distinguishes EoR models
- Analysis for other triangles
- Higher multipoles

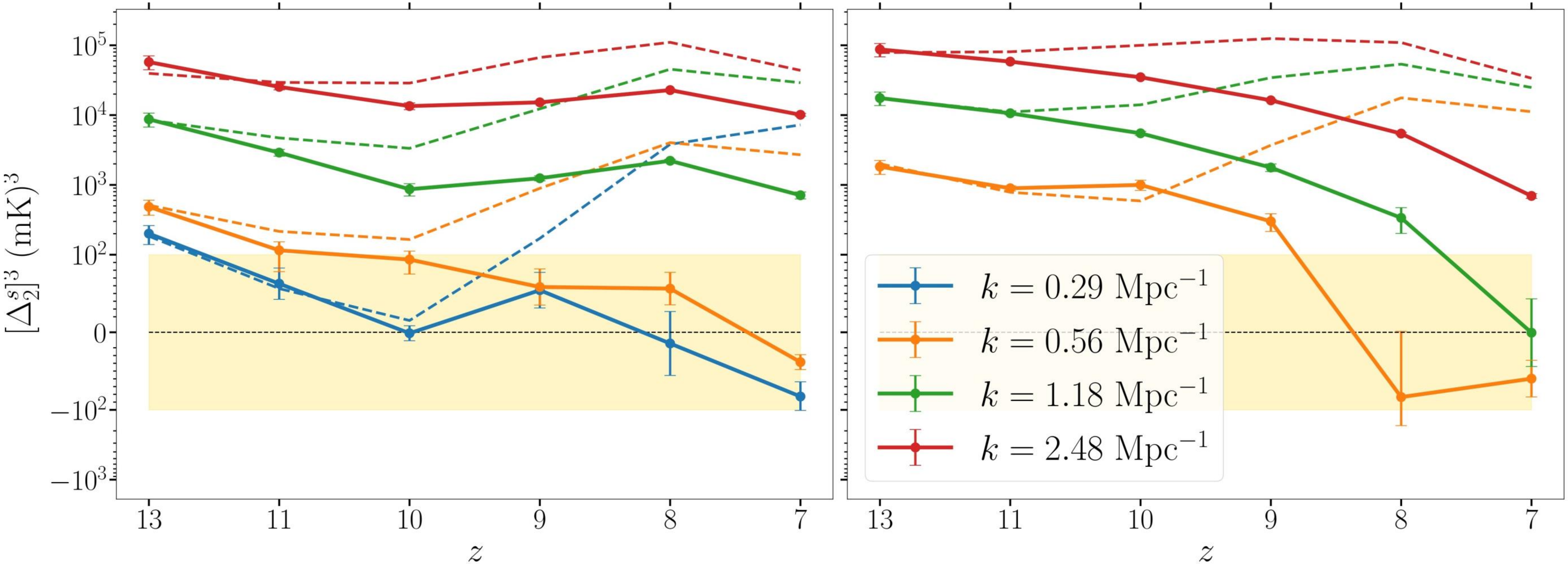
THANK YOU

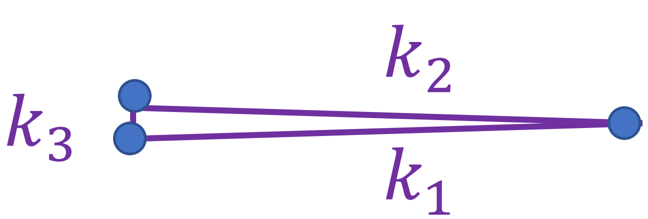
- Millenium simulation



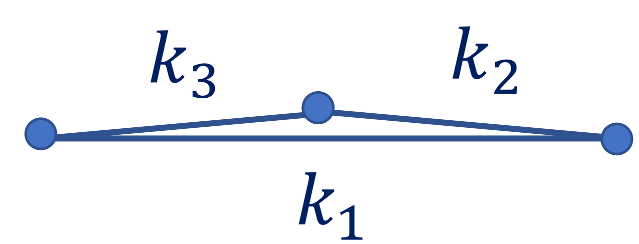
$\bar{x}_{\rm HI}$
0.99
0.93
0.86
0.73
0.50
0.15
$[\Delta_0^s]^3 \approx [\Delta_0^s]^3_M + [\Delta_0^s]^3_I$
nonlinear matter distribution
Ionization term
$[\Delta_0^s]^3$ (mK)^3
10^5
10^4
10^3
10^2
0
-10^2
-10^3
Squeezed
$z = 13$
$z = 11$
$z = 10$
$z = 9$
$z = 8$
$z = 7$
cMpc
T_b (mK)

Squeezed | *Stretched*





$(k_1 \approx k_2$ & $k_3 \approx 0)$

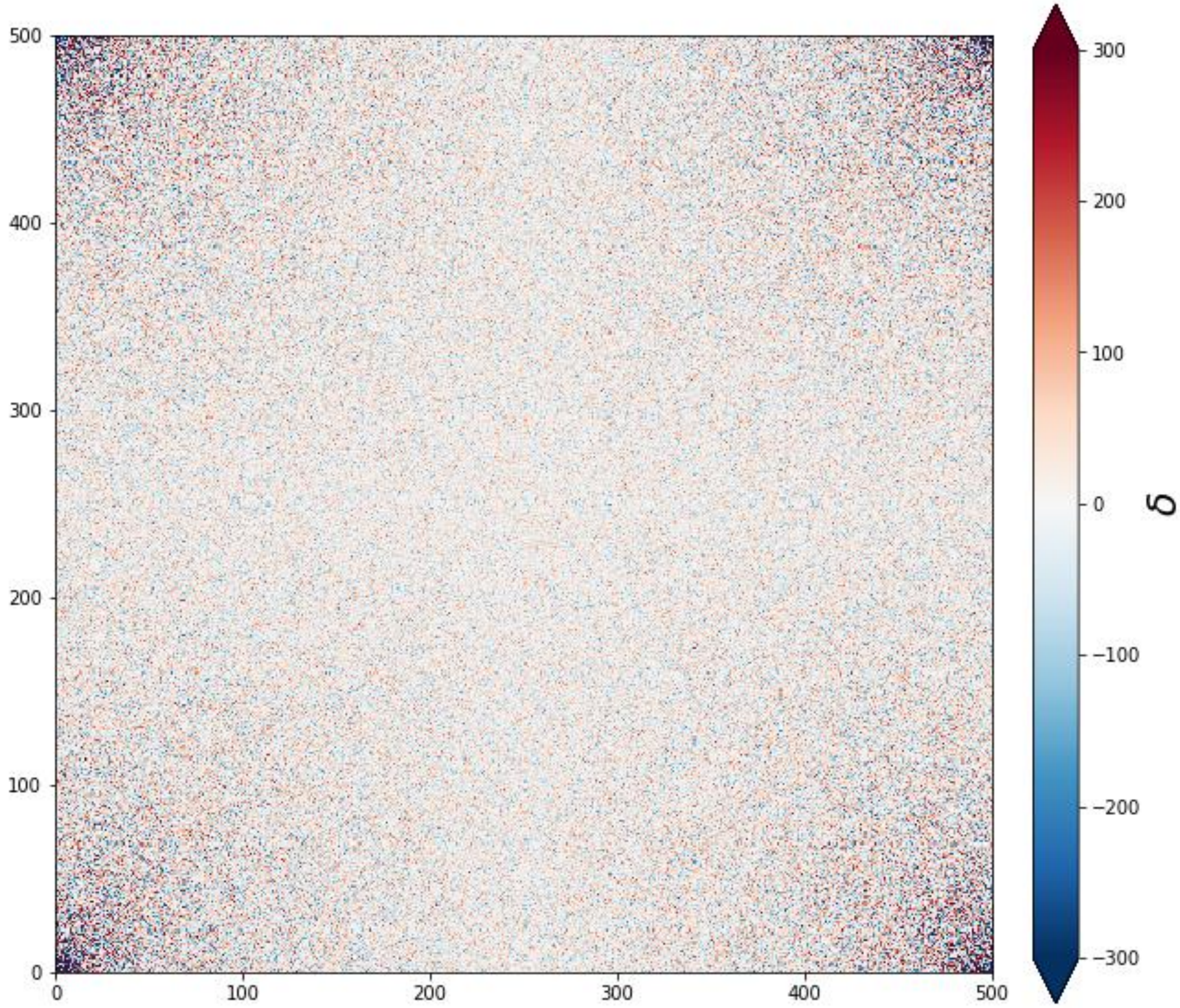


$(k_1 \approx 2k_2$ & $k_3 \approx k_2)$

Interpretation

- Non-Gaussianity is small at large length scales at high redshifts because HII regions are small in size at very early times (small fluctuations)
- Bispectrum increases with k_1 as larger modes becomes sensitive to small HII regions
- Bispectrum increases with time as non-Gaussinity increases with progress of EoR; maximize at around $\bar{x}_{HI} \approx 0.5$ when HII regions are sufficiently large; BS again decreases as NG is due to HI regions
- At smaller length scales (largest k_1), BS decreases with time due to increase in size and number density of HII region
- Bispectrum is –ve if NG is driven by x_{HI} field; and positive is NG is driven by matter density fluctuations (Hutter et al. 2020; Majumdar et al. 2018)

Uncorrelated GRF

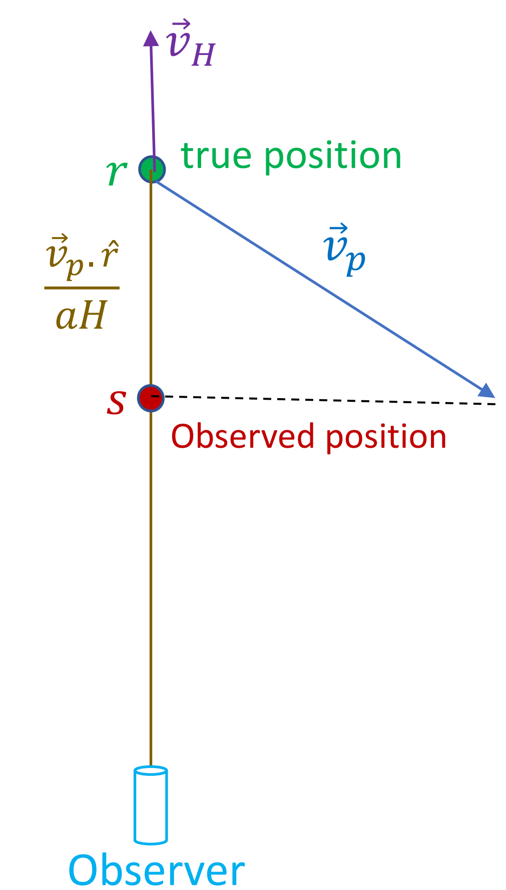


Redshift Space Distortion

- Distribution is observed in **redshift space** not in real space

$$z = z_H \oplus z_{pec} = \left[(1+z_H) \times (1+z_{pec})\right] - 1 \qquad ; \quad \left(a_e = \frac{\lambda_e}{\lambda_o - \lambda_e} = \frac{1}{1+z} \right)$$

Observations $\begin{pmatrix} \theta \\ \phi \\ z \end{pmatrix}$ → 3D mapping: $s = \int_0^z \frac{c\, dz'}{H(z', \Omega)}$ → *Distances* $\begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix}$

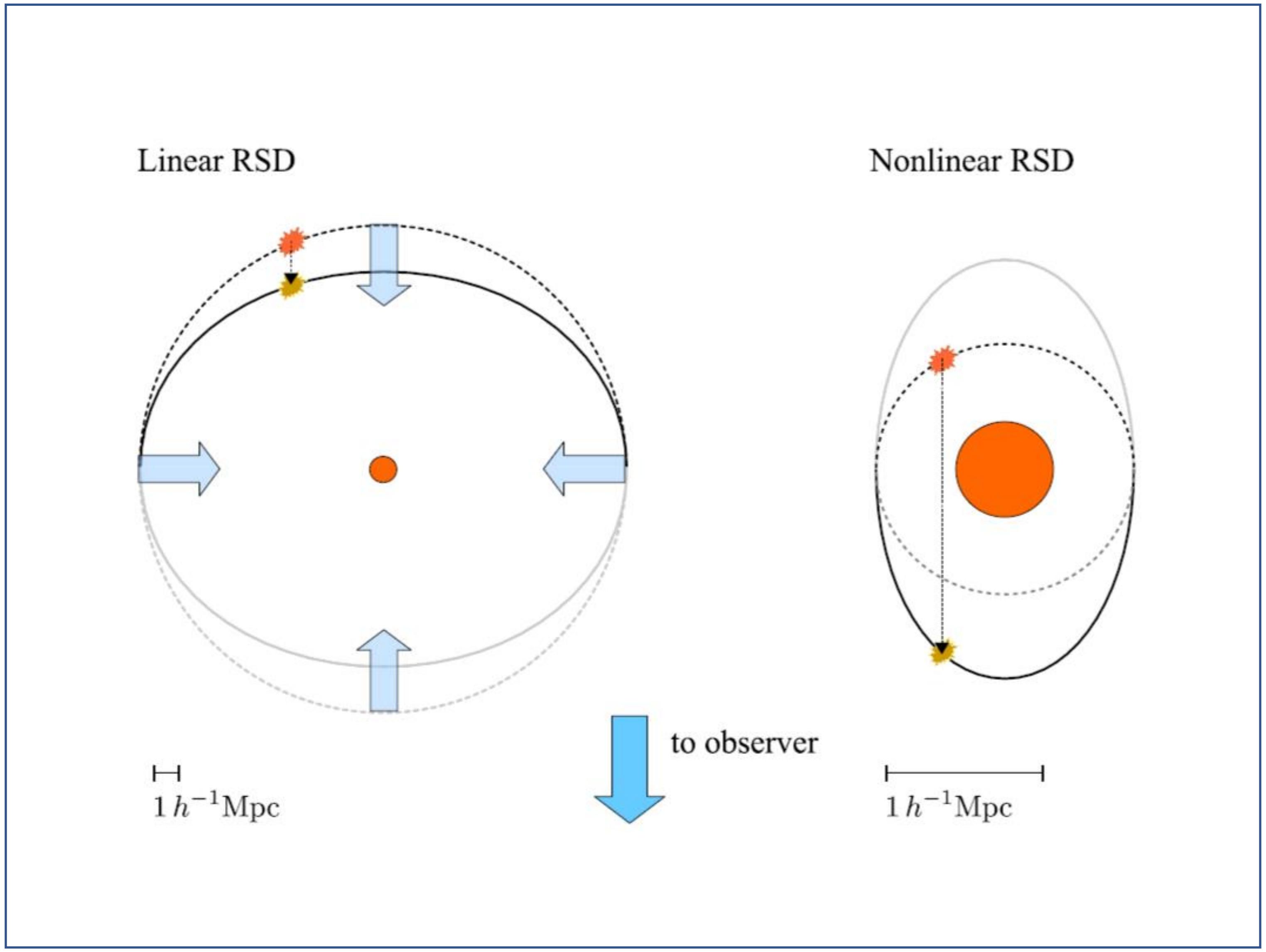


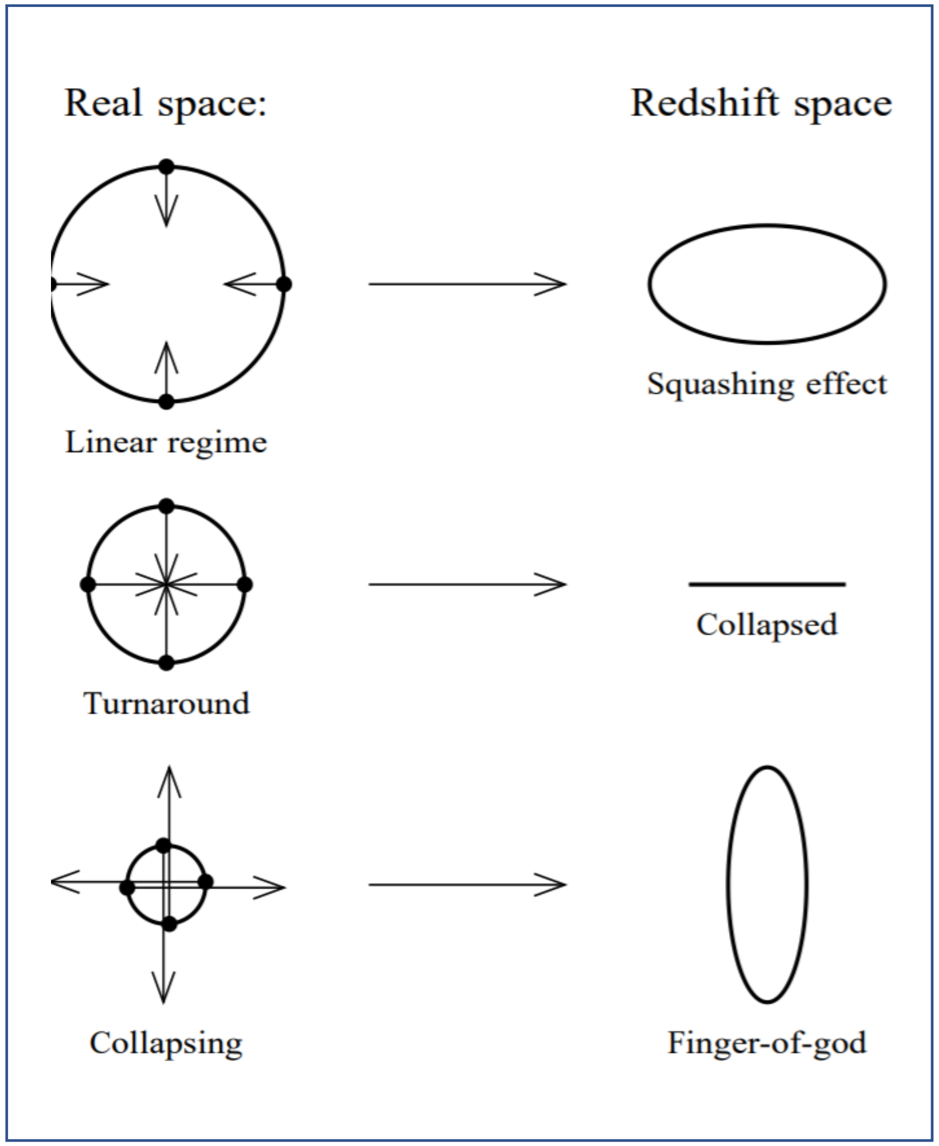
- True and observed distances

$$s = r + \frac{\vec{v}_p.\hat{r}}{a\,H}$$

LoS component of peculiar velocity

Redshift Space Distortion





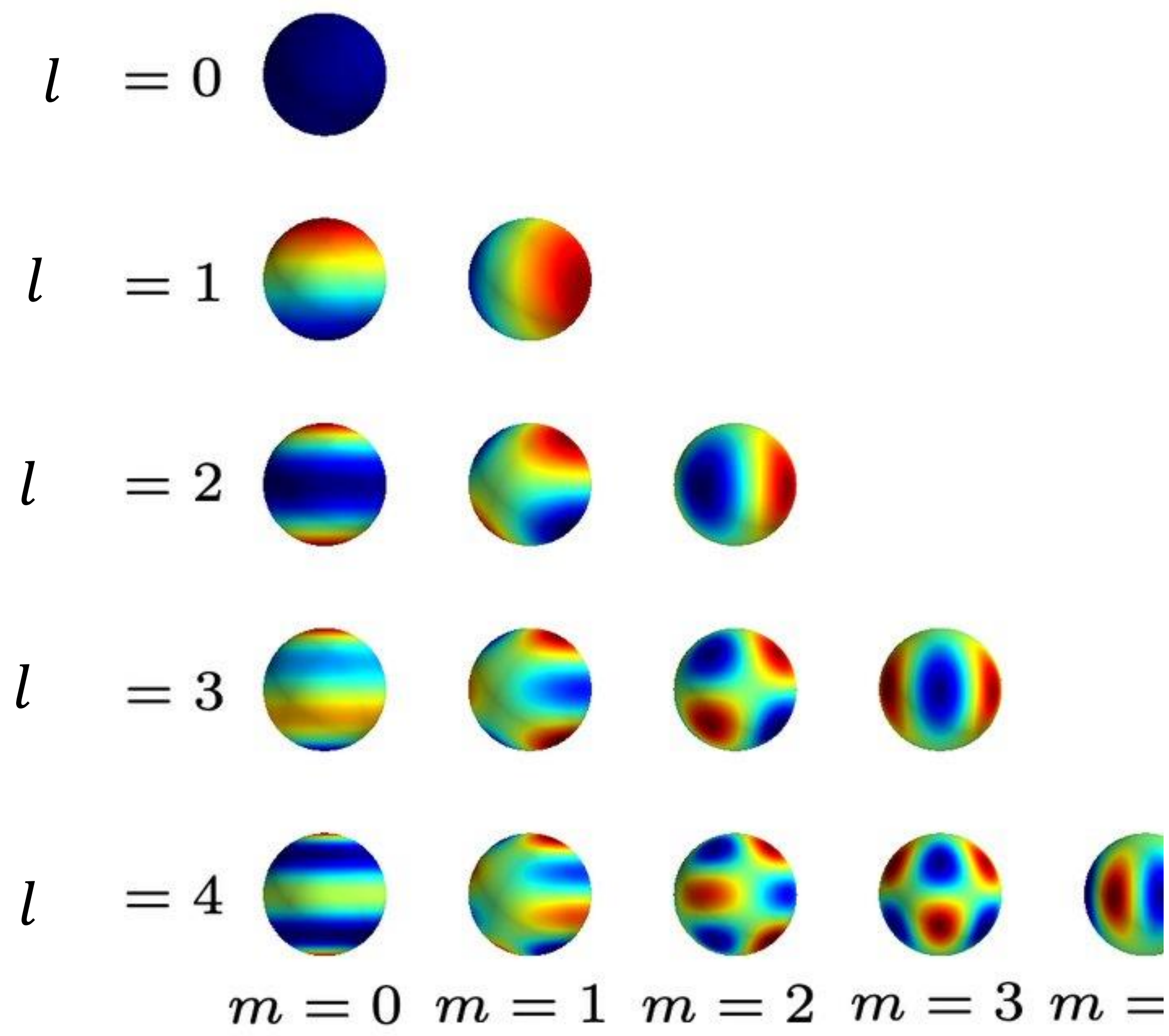
- Number density at center increases (for over-dense regions)
- Overall effect introduces "apparent anisotropy"
- However, distribution still posses isotropy in plane perpendicular to LoS

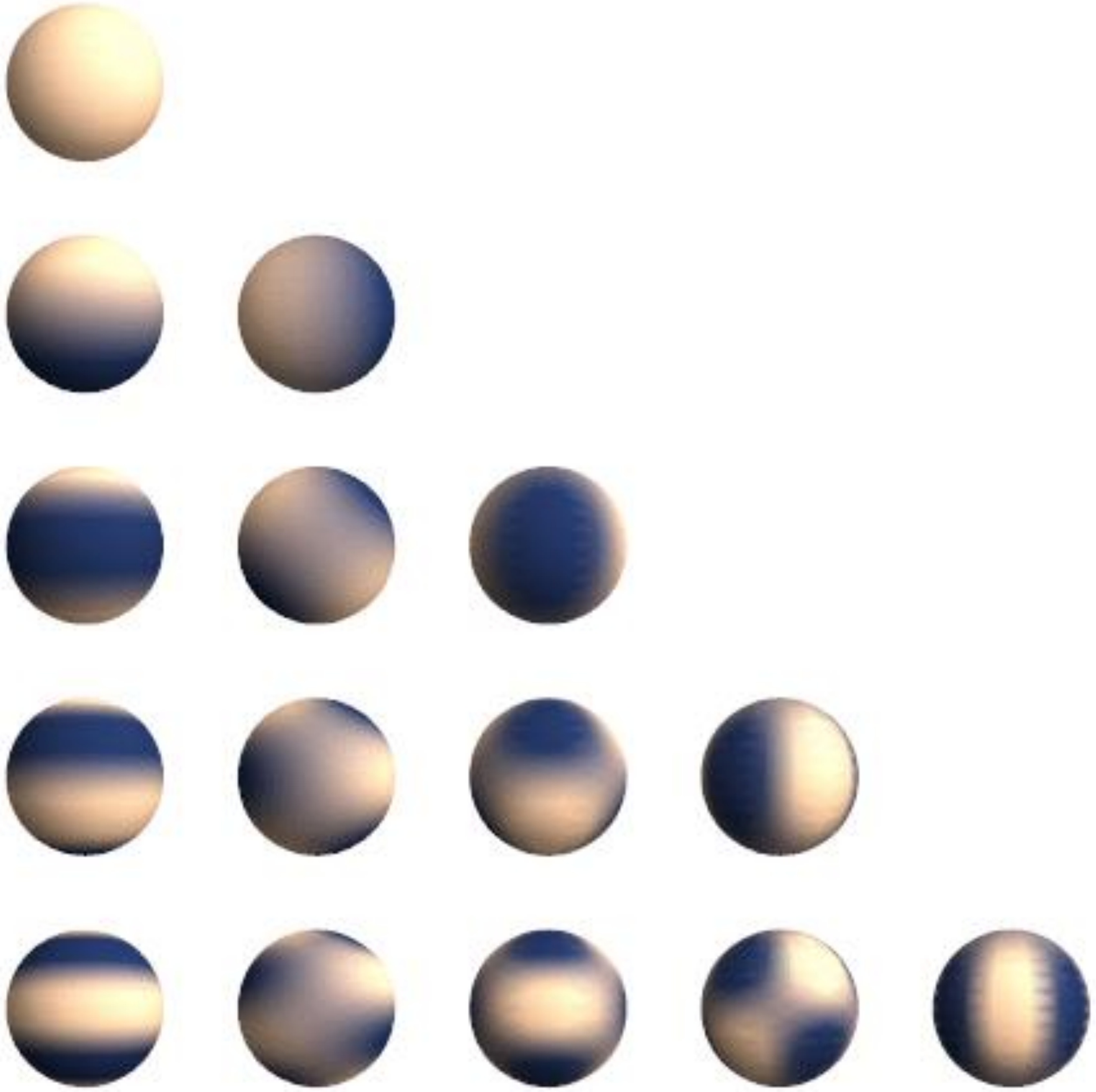
Brightness Temperature Fluctuations

$$\delta T_b(\hat{n}, z) = \left(1 - \frac{T_\gamma}{T_s}\right) \bar{T}(z) \frac{\rho_{HI}}{\bar{\rho}_H} \left[1 - \frac{1}{aH} \frac{\partial v}{\partial r}\right]$$

$$\bar{T}(z) = 4mK\ (1+z)^2\ \left(\frac{\Omega_b h^2}{.02}\right) \left(\frac{.7}{h}\right) \frac{H_0}{H(z)}$$

$$Y_l^m$$





Source: wikipedia