# Advanced 21-cm Cosmology Workshop (2023)

NISER Bhubaneswar,
19th December, 2023

# A method to simultaneously determine the reionization history and power spectrum

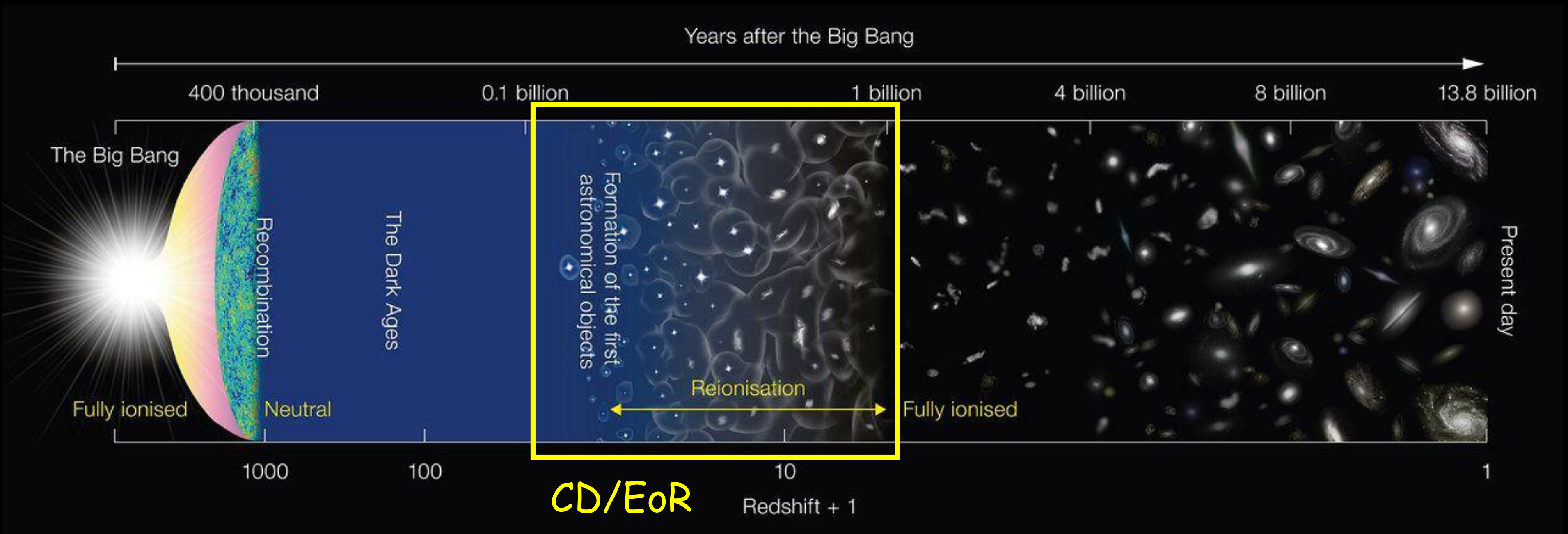

by

**Suman Pramanick**

In collaboration with

Somnath Bharadwaj,
Rajesh Mondal &
Asif Elahi

# Cosmic History



GMRT

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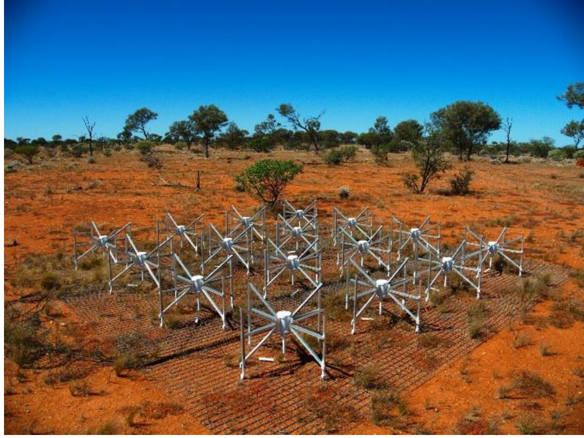

MWA

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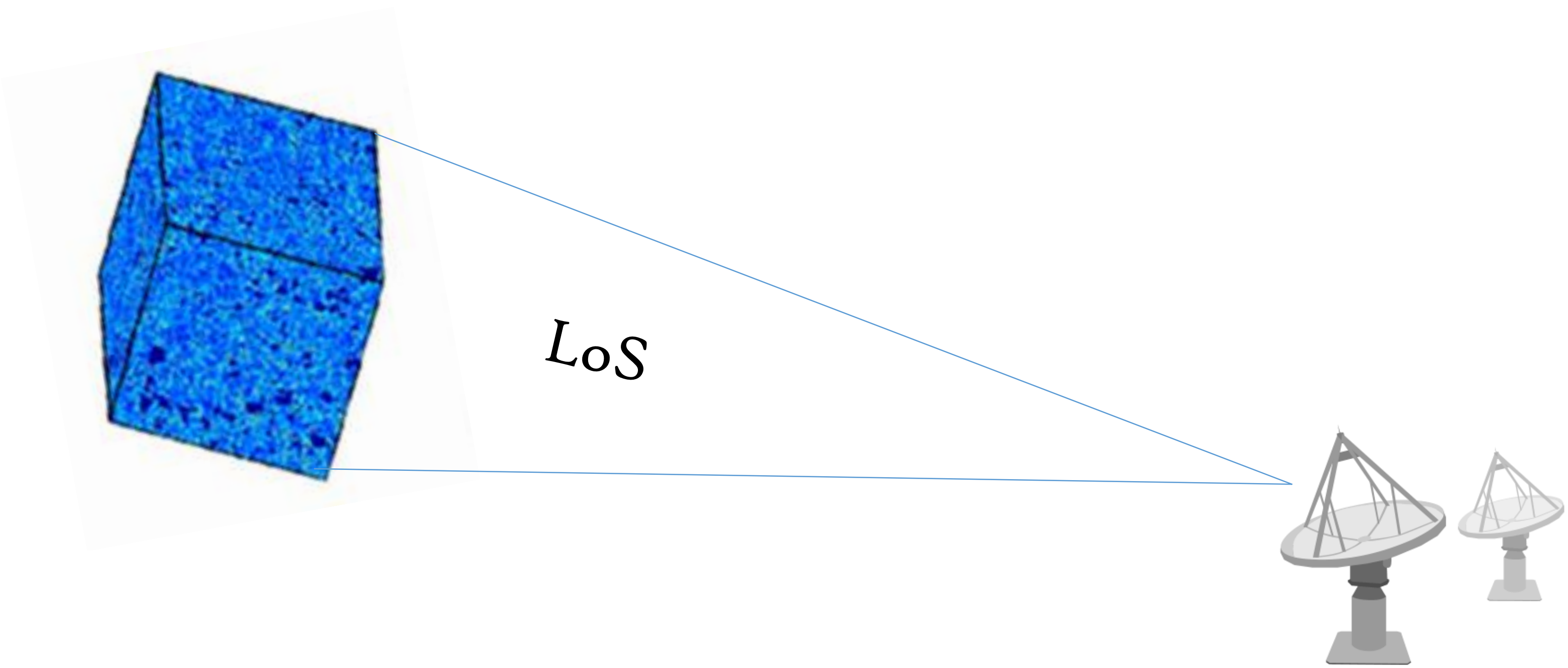
# Two important line of sight (LoS) effects

- Redshift space distortion (RSD)
- Lightcone effect

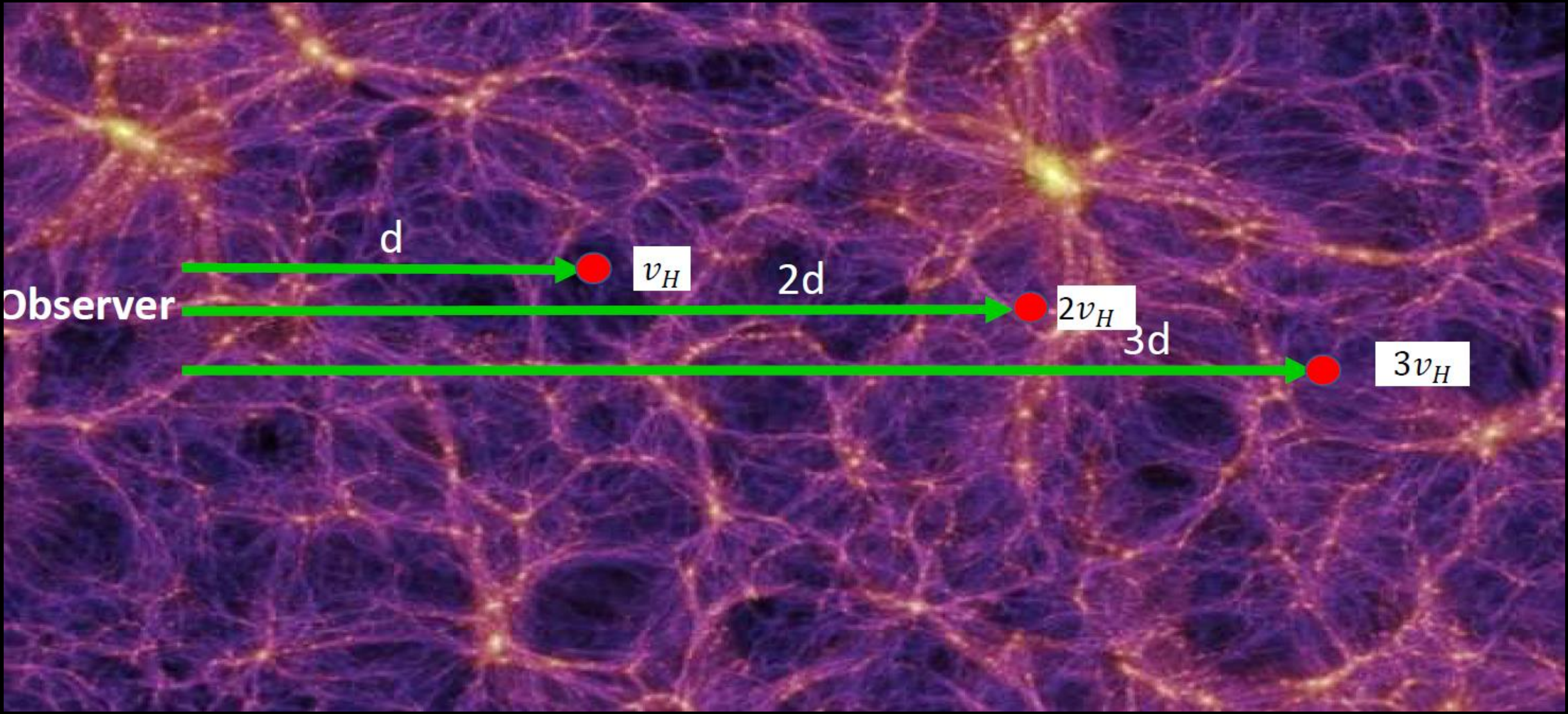


# Redshift Space Distortion (RSD)

Redshift-space distance $\boldsymbol{s}$ can be calculated and comes out as

$$\boldsymbol{s} = \boldsymbol{r} + \frac{\boldsymbol{v}_p . \boldsymbol{n}}{aH(a)}$$

Where, $\boldsymbol{v}_p$ is the peculiar velocity and $\boldsymbol{n}$ is the line of sight direction, $a$ is the scale factor and $H(a)$ is the Hubble parameter



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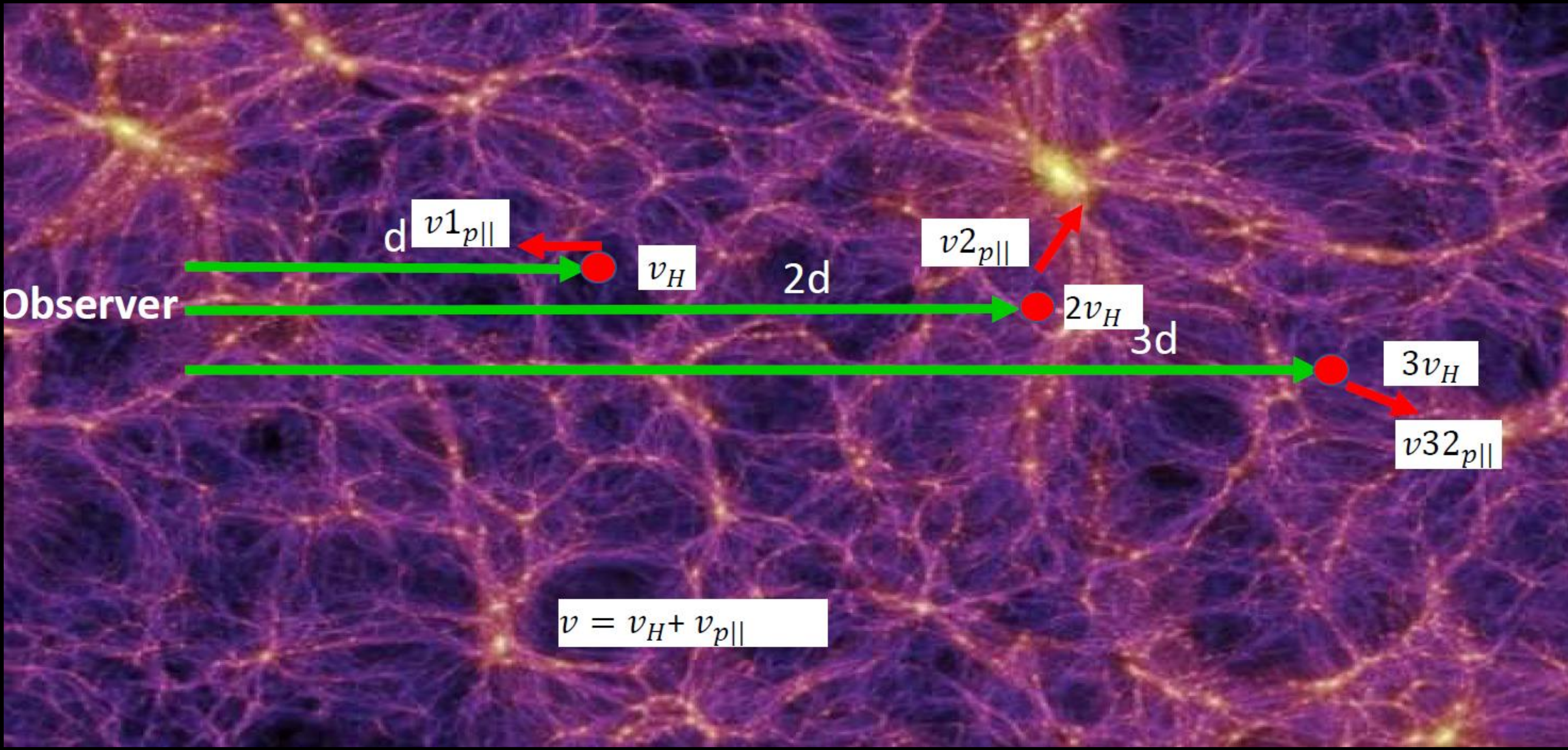

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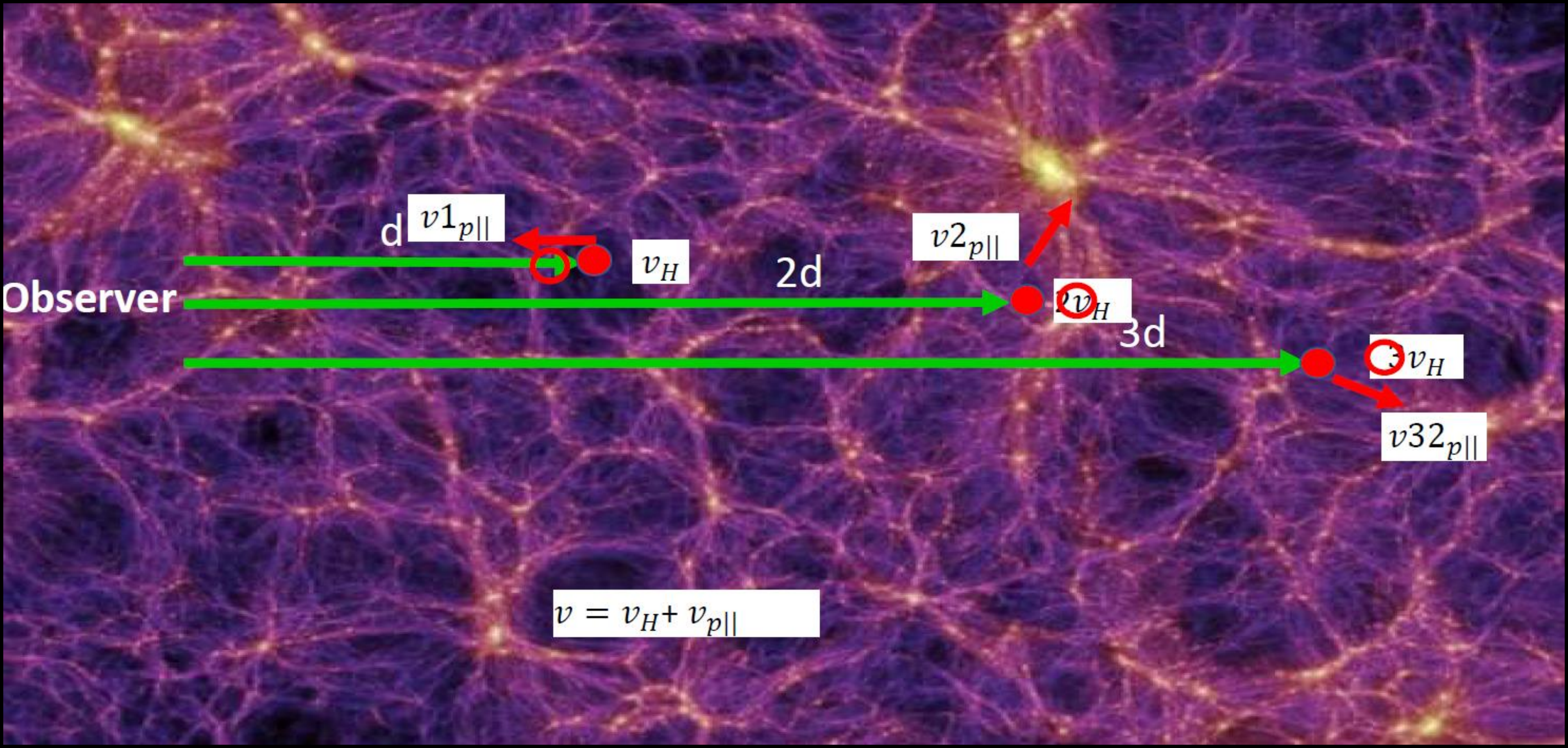
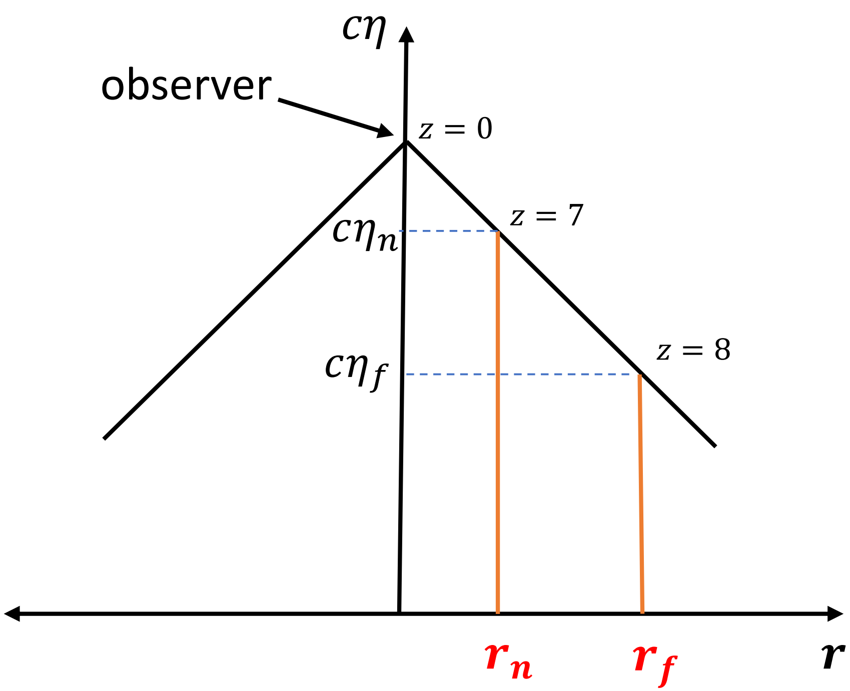


# Lightcone effect




- It is the fact that our view of the universe is restricted through a backward light-cone which can be written as

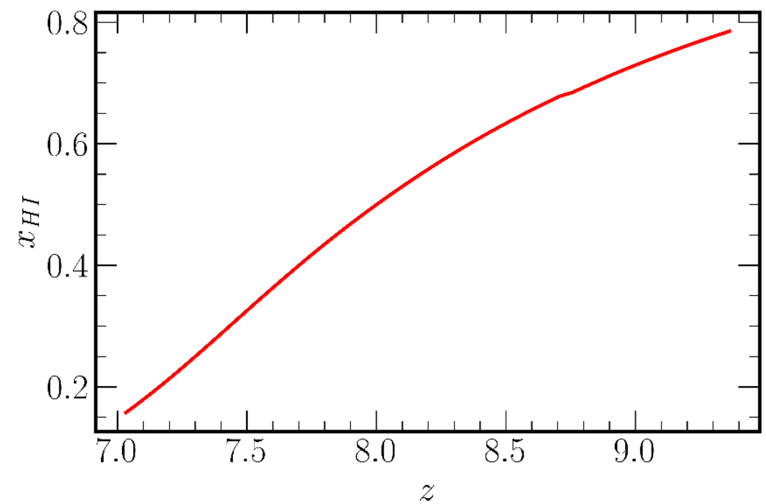
$$r = c(\eta_0 - \eta)$$

Comoving distance ($r$)

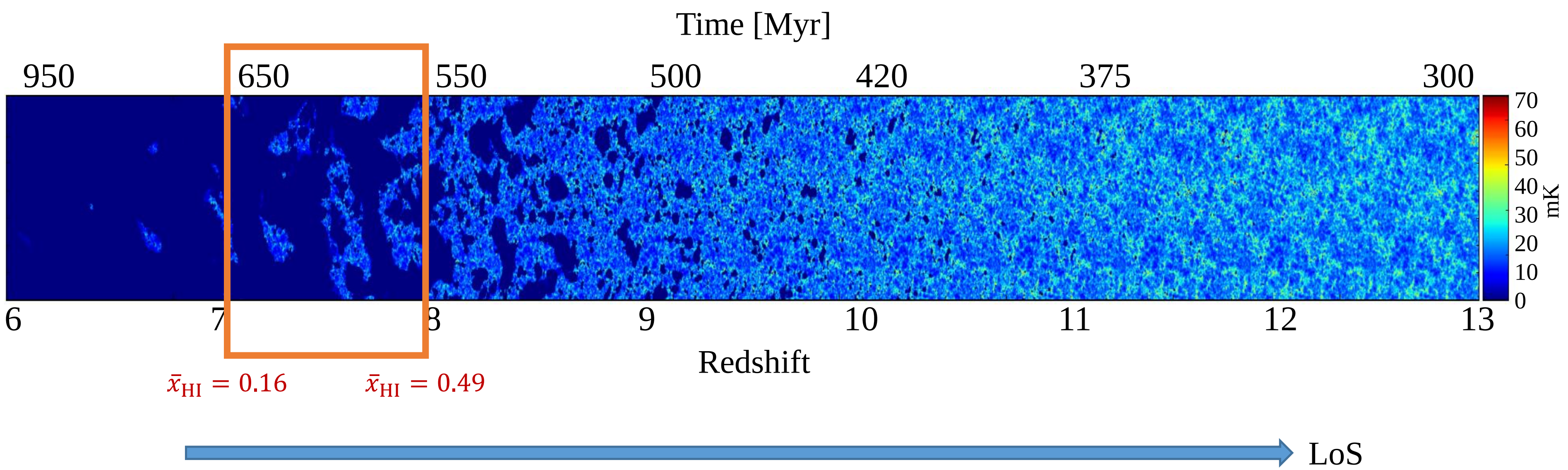
Present epoch ($\eta_0$)

Epoch at the source ($\eta$)

- Lightcone effect is significant during EoR as mean neutral hydrogen fraction $\bar{x}_{\rm HI}$ changes rapidly during this epoch.



# Epoch of Reionization (EoR)

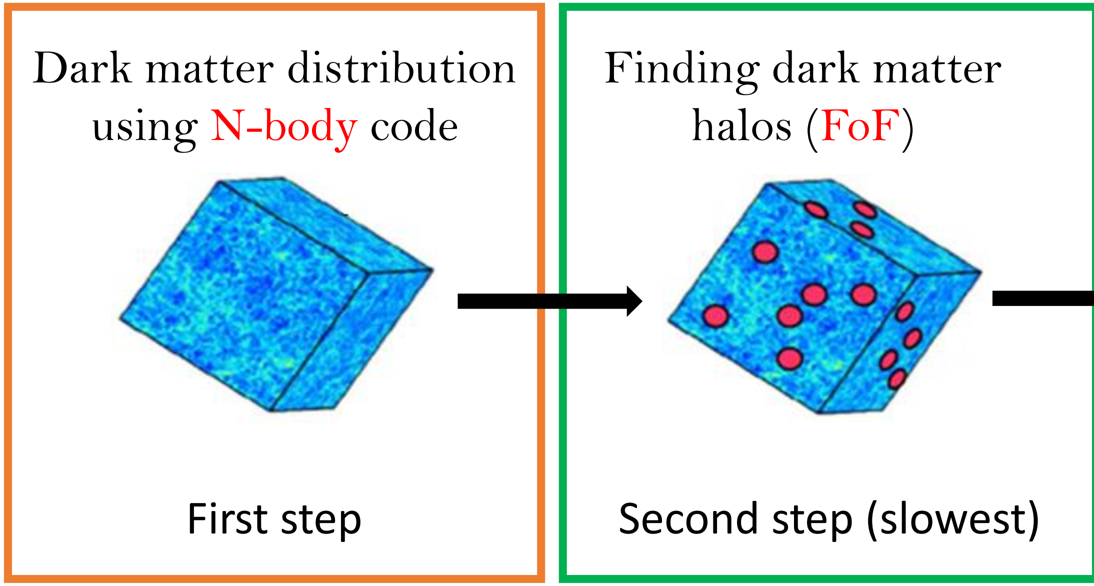


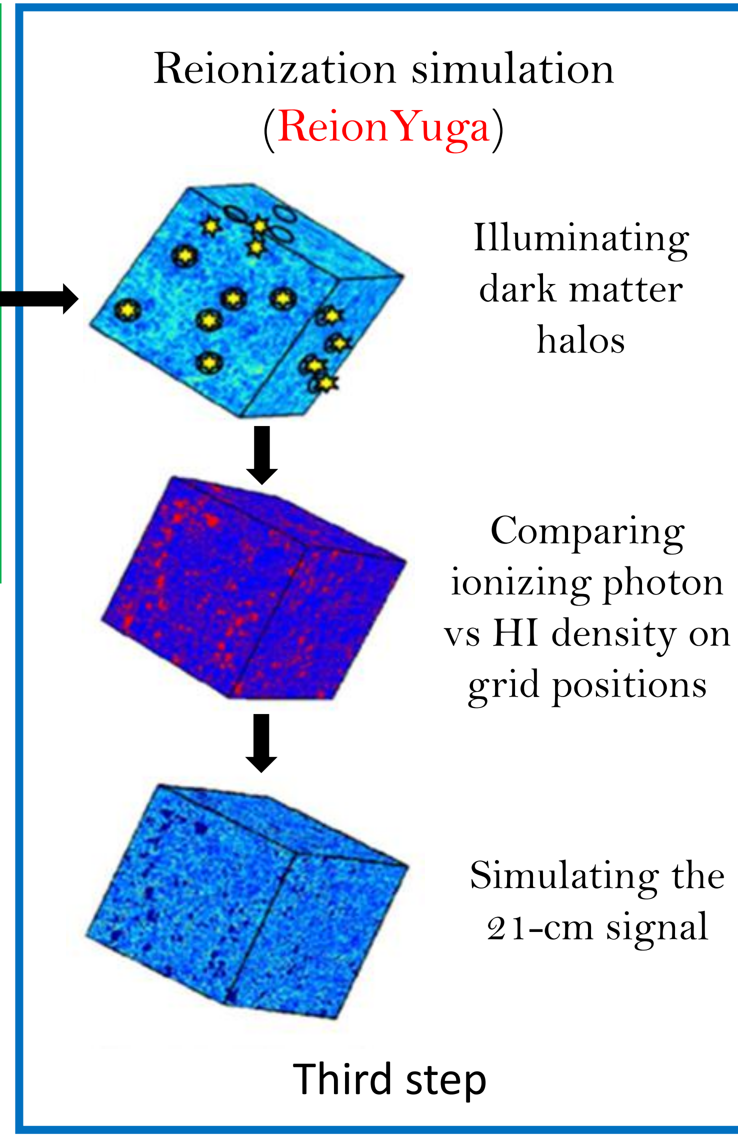

- A simulated box of comoving size $(286.7\ Mpc)^3$, centered around redshift $z_c = 7.46$ extends from $z =$ 7.03 to 7.91 and the $x_{\mathrm{HI}}$ changes from 0.16 to 0.49 respectively.

- Neutral fraction, statistical properties of HI fluctuations changes substantially in the redshift range

- A simulated cube that captures redshift evolution of the signal termed as **'light cone'** simulation.

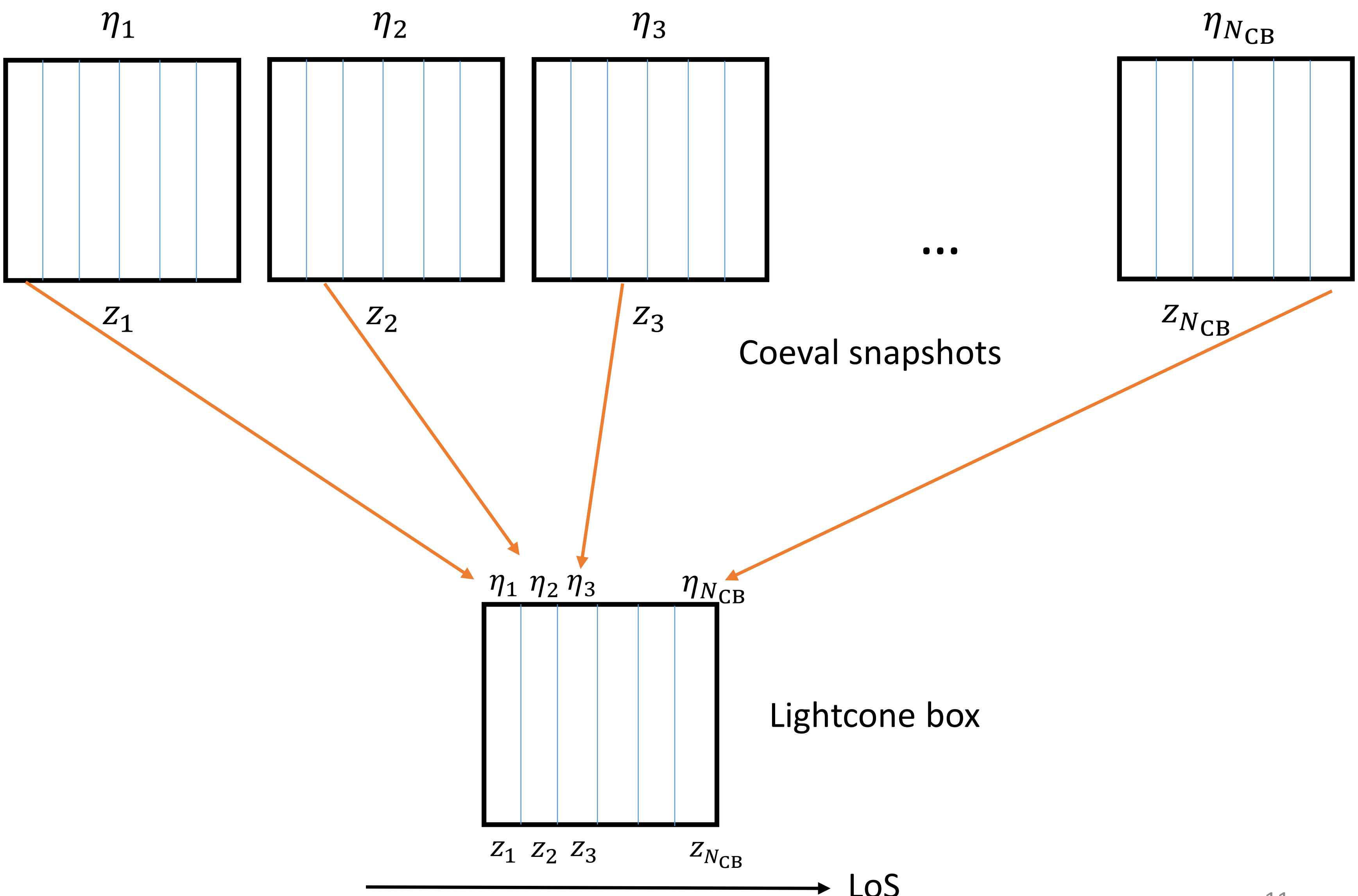
# Simulating the EoR (Coeval Box/CB)




- Particle-mesh N-body: Dark matter only simulation, box size is limited by the RAM.
- FoF: Take longest time to run.
- ReionYuga: Closely follows the excursion set formalism of Choudhury et al. (2009)

# Simulating the EoR lightcone



- R.M. Thomas, S. Zaroubi, B. Ciardi, A.H. Pawlik, P. Labropoulos, V. Jeli´c et al., Fast large-scale reionization simulations, Monthly Notices of the Royal Astronomical Society 393 (2009) 32.

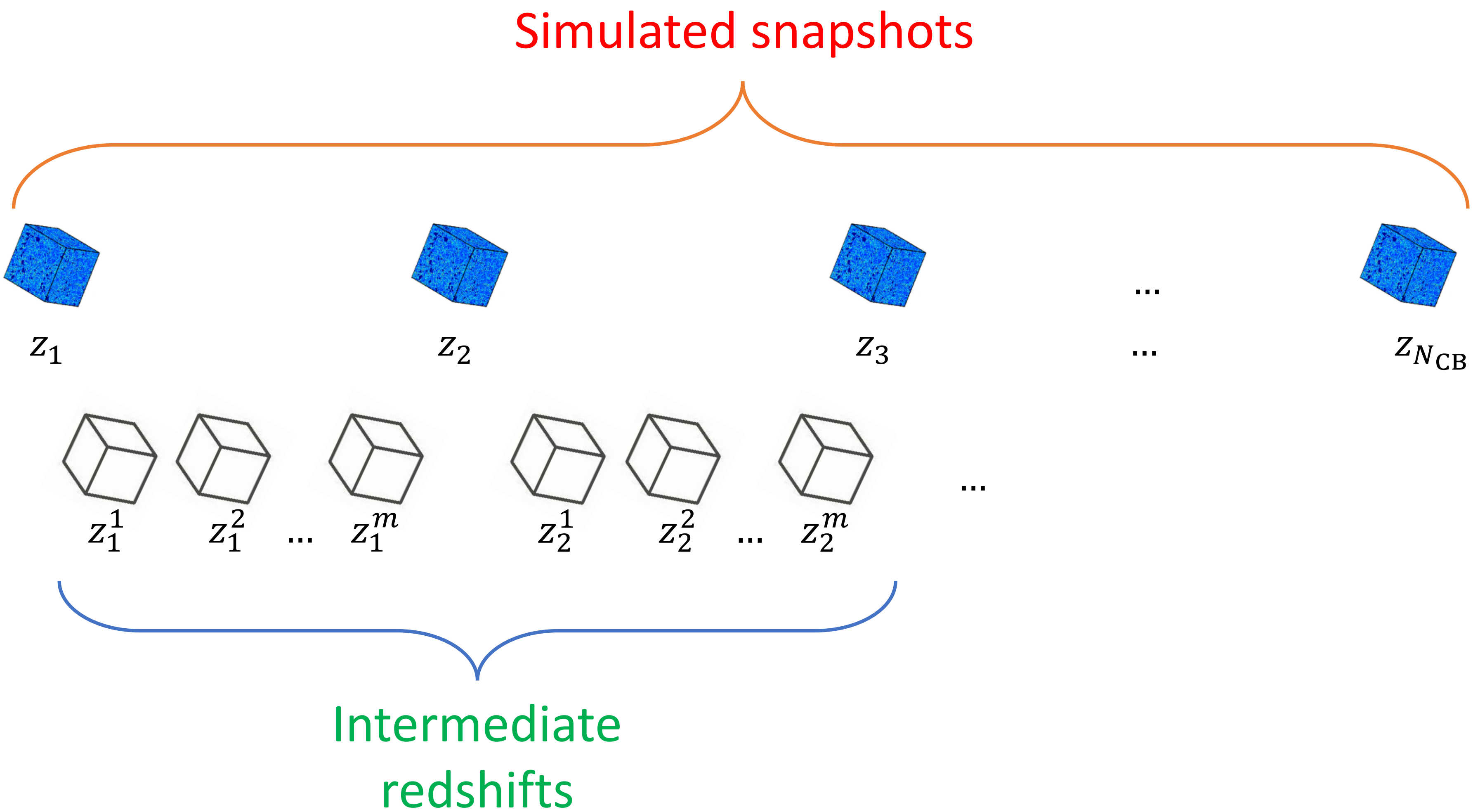
- K.K. Datta, G. Mellema, Y. Mao, I.T. Iliev, P.R. Shapiro and K. Ahn, Light-cone effect on the reionization 21-cm power spectrum, Monthly Notices of the Royal Astronomical Society 424 (2012) 1877.

- K. Zawada, B. Semelin, P. Vonlanthen, S. Baek and Y. Revaz, Light-cone anisotropy in the 21 cm signal from the epoch of reionization, Monthly Notices of the Royal Astronomical Society 439 (2014) 1615.

- K.K. Datta, H. Jensen, S. Majumdar, G. Mellema, I.T. Iliev, Y. Mao et al., Light cone effect on the reionization 21-cm signal–ii. evolution, anisotropies and observational implications, Monthly Notices of the Royal Astronomical Society 442 (2014) 1491.

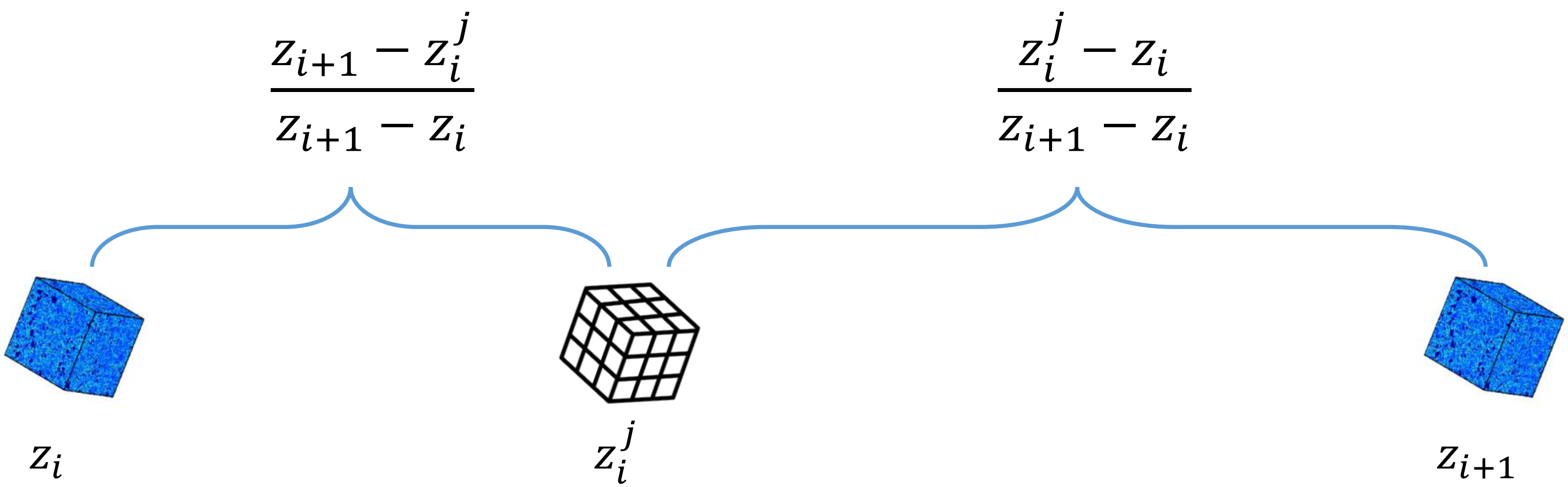
- X. Zhao, Y. Mao, C. Cheng and B.D. Wandelt, Simulation-based inference of reionization parameters from 3d tomographic 21 cm light-cone images, The Astrophysical Journal 926 (2022) 151.

# Interpolation




S. Pramanick et al. (2023)

# Interpolating Matter and Halo fields

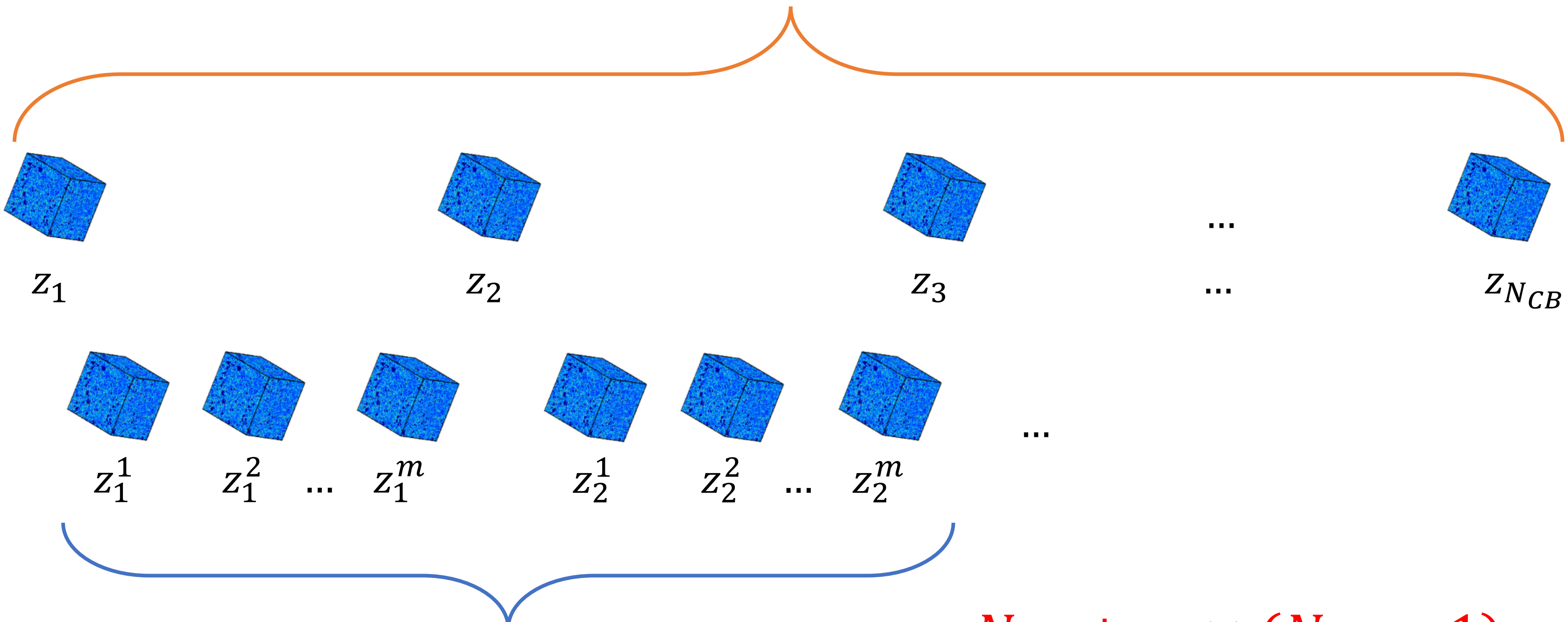



Interpolation after gridding

S. Pramanick et al. (2023)

# Increased number of snapshots

Simulated snapshots

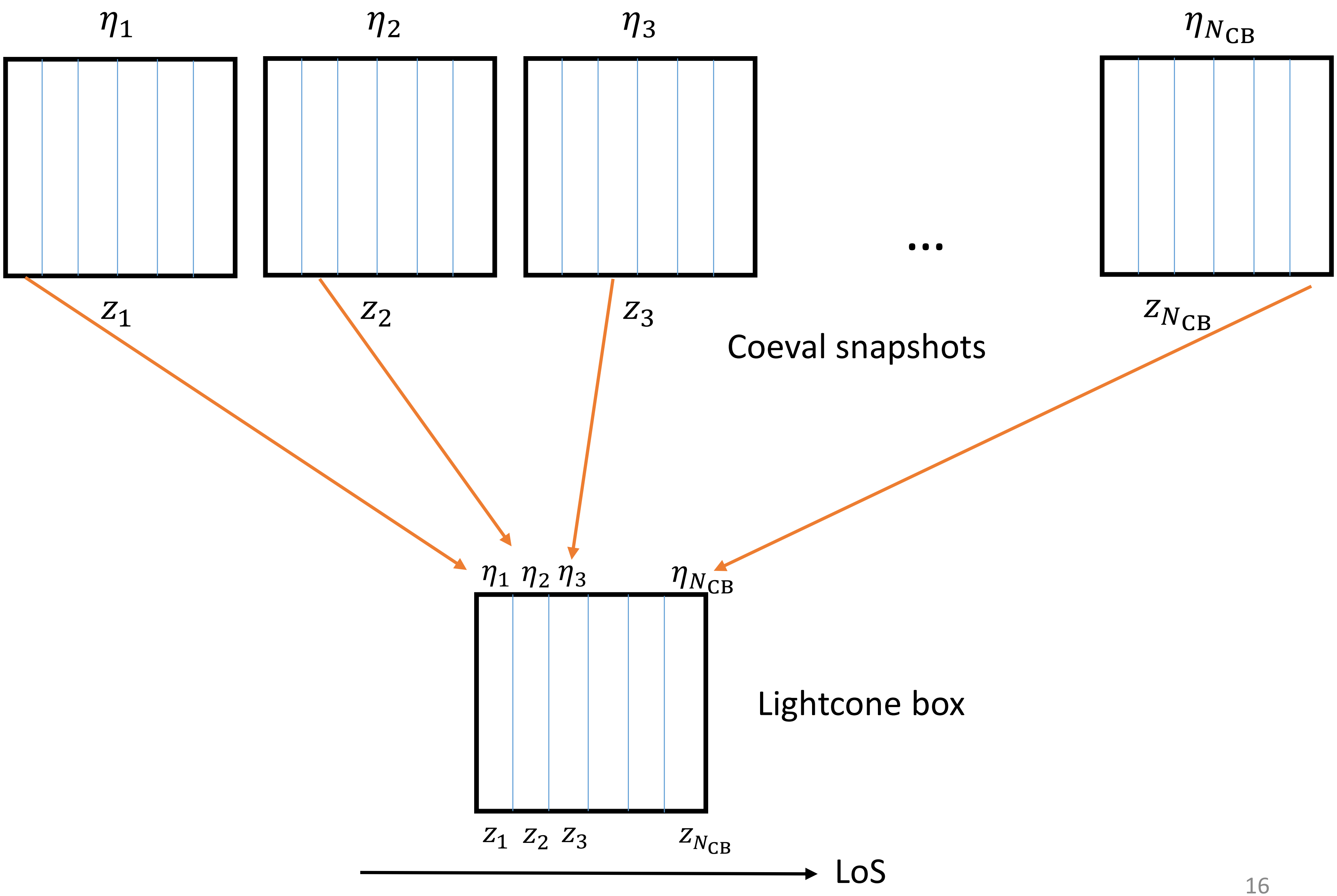



Intermediate redshifts

$$\underbrace{N_{\mathrm{CB}}}_{\text{accurate}} + \underbrace{m \times (N_{\mathrm{CB}} - 1)}_{\text{interpolated}}$$

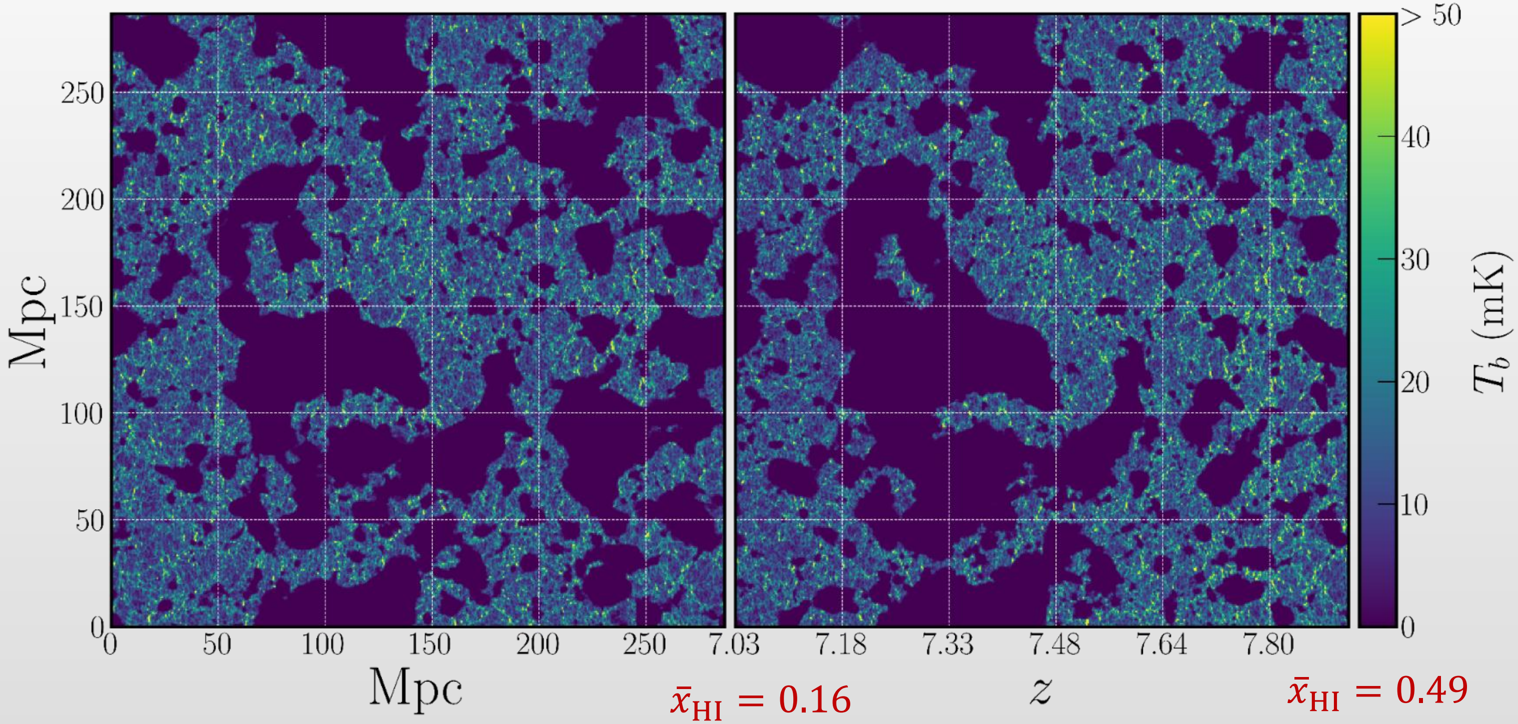
S. Pramanick et al. (2023)

# Simulating the EoR lightcone



Coeval box

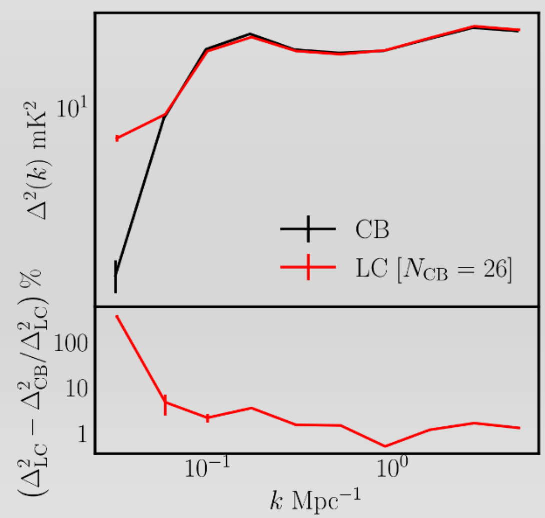
Lightcone box



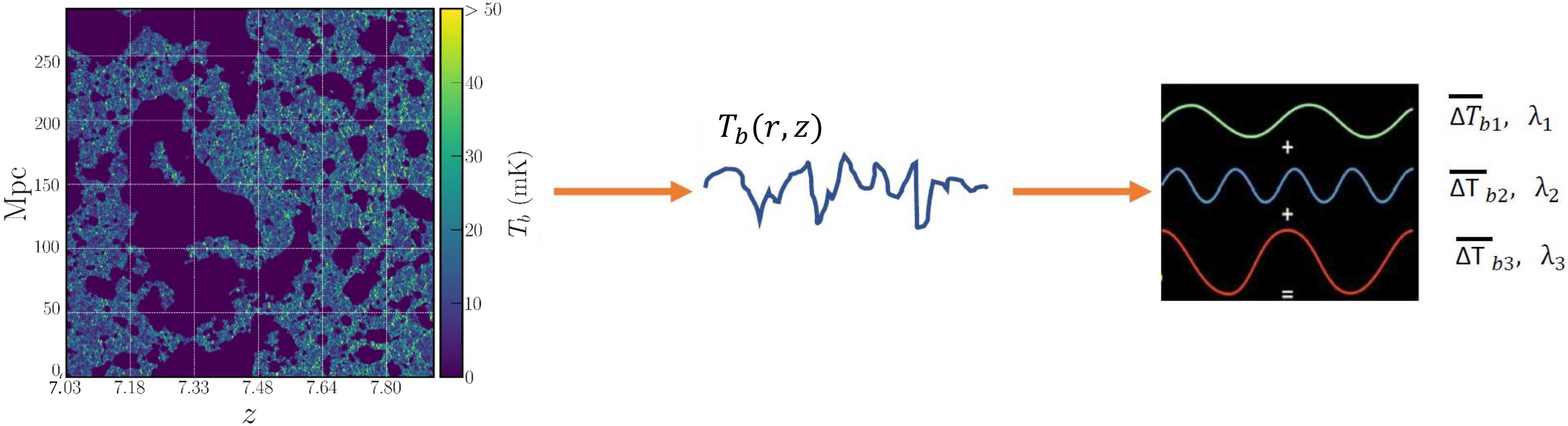

$\bar{x}_{\mathrm{HI}} = 0.16$

$\bar{x}_{\mathrm{HI}} = 0.49$

- Central redshift $z_c = 7.46$
- Central frequency $\nu_c = 167.9$ MHz
- Central commoving distance $\mathrm{r_c} =$ 8986.4 Mpc
- Bandwidth = 17.3 MHz
- Box size = $(286.7\ \mathrm{Mpc})^3$



# 3D Power spectrum



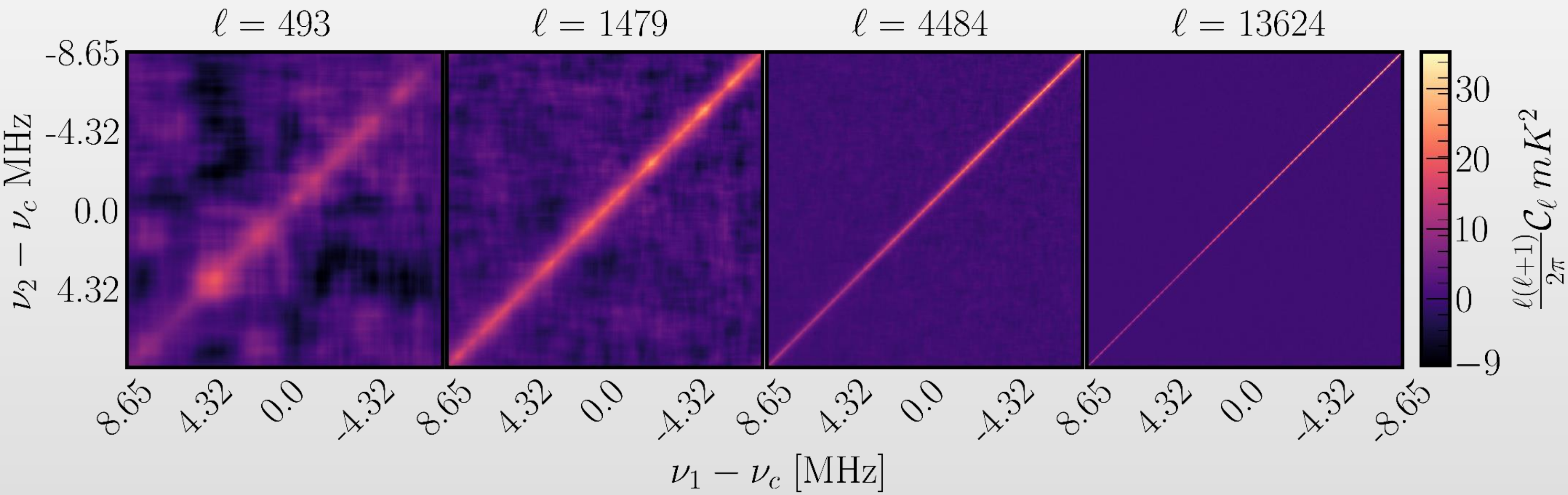

- 3D spherically averaged power spectrum is defined as:

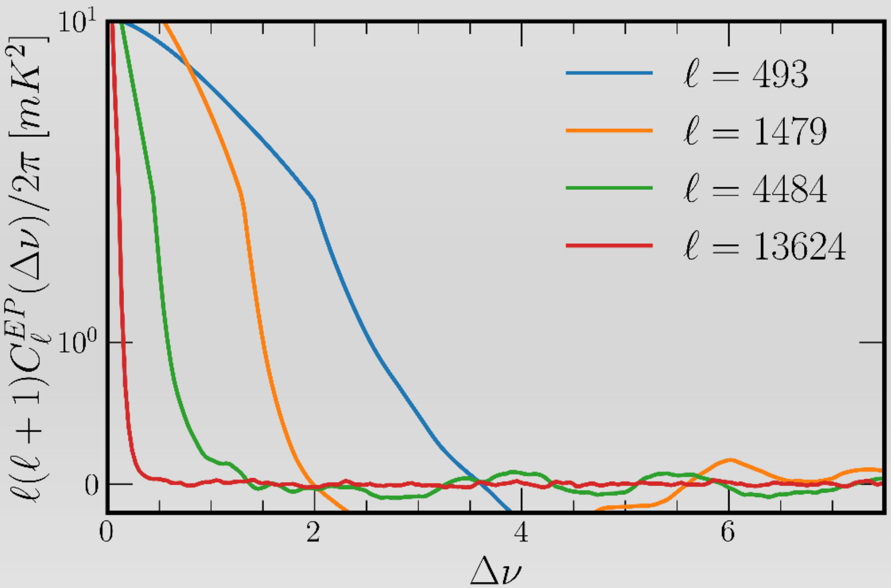
$$P(\boldsymbol{k}) = V^{-1} < \widetilde{T_b}(\boldsymbol{k})\, \widetilde{T_b}(-\boldsymbol{k}) >$$

- It assumes statistical homogeneity and imposes periodicity on the signal which cannot be justified in the presence of LC effect (Trott 2016).

- In contrast Multifrequency Angular Power Spectrum (MAPS) does not have any such intrinsic assumptions (Mondal et al. 2018).

# Multifrequency Angular Power Spectrum (MAPS)



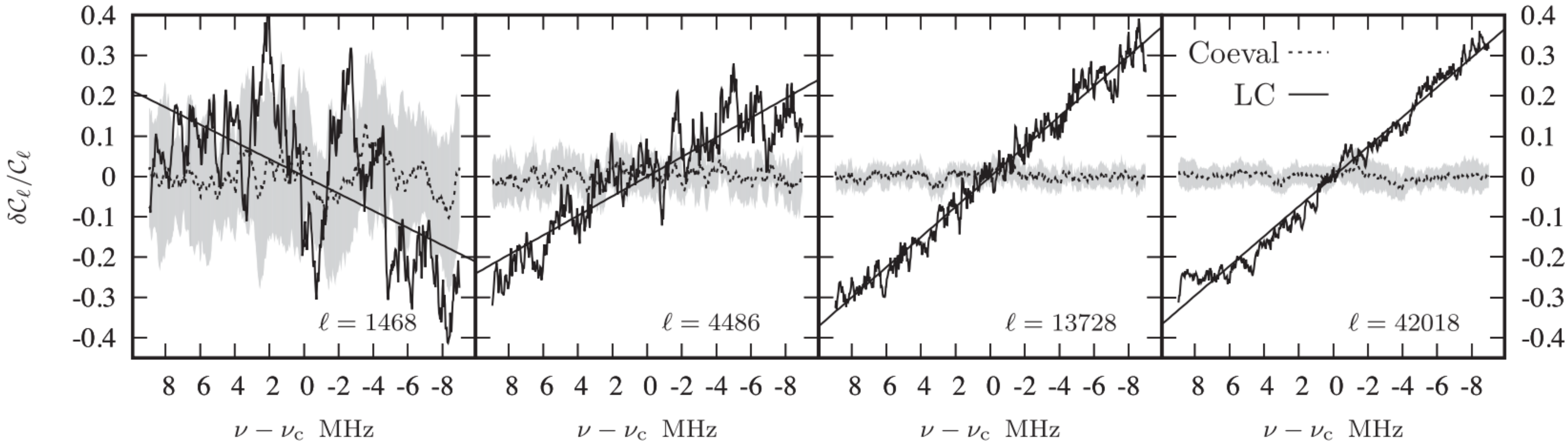




- MAPS is defined as

$$C_\ell(\nu_1, \nu_2) = C_{2\pi U}(\nu_1, \nu_2) = \Omega^{-1} < \tilde{T}_{b2}(\boldsymbol{U}, \nu)\ \tilde{T}_{b2}(-\boldsymbol{U}, \nu) >$$

- $\Omega$ is solid angle with respect to the observer

# $\mathcal{C}_\ell(\nu,\nu)$:



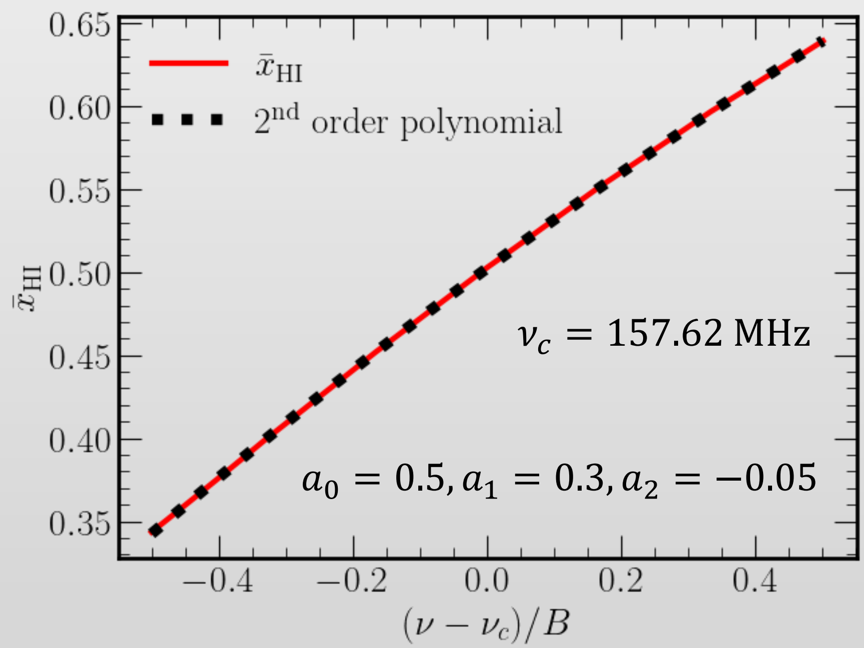

- Diagonal elements of $\mathcal{C}_\ell(\nu_1,\nu_2)$ shows systematic increase with redshift for LC simulation, where $x_{\rm HI}$ also increase with $z$.
- Similar behavior is absent in coeval simulation, where $\bar{x}_{\rm HI}$ remains constant.
- We can assume the evolution of $\mathcal{C}_\ell(\nu_1,\nu_2)$ along the LoS is arising entirely due to the evolution of $\bar{x}_{\rm HI}$.
- The homogeneous and isotropic statistical fluctuations can be quantified using power spectrum.

# The model

$$C_\ell(\nu_1, \nu_2) = \underbrace{\bar{x}_{\rm HI}(\nu_1)\, \bar{x}_{\rm HI}(\nu_2)}_{\text{non ergodic}}\, \underbrace{C^E_\ell(\nu_1, \nu_2)}_{\text{ergodic}}$$

- The ergodic part can be modeled using monopole $P_0(k)$ and quadrupole $P_2(k)$ moments of power spectrum
- The non ergodic part can be modeled by modeling $\bar{x}_{\rm HI}$.

# Non ergodic part: $\bar{x}_{\rm HI}(\nu_1)\,\bar{x}_{\rm HI}(\nu_2)$




- We model $\bar{x}_{\rm HI}$ using a second order polynomial

$$\bar{x}_{\rm HI} = a_0 + a_1 \frac{\nu - \nu_c}{B} + a_2 \left(\frac{\nu - \nu_c}{B}\right)^2$$

Where, $B$ is the bandwidth of the observation.

# Ergodic part: $C_\ell^E(\nu_1, \nu_2)$

- In the presence of RSD power spectrum has multipole contributions, see eg. [S. Majumdar et al. (2013)]

$$P(k,\mu) = \sum_{l=\text{even}} \wp_l(\mu) P_l(k)$$

Where $\mu = \frac{\boldsymbol{k}.\boldsymbol{n}}{k} = \frac{k_\parallel}{k}$ and $\wp_l(\mu)$ are Legendre Polynomials

- Considering up to quadrupole moment

$$P(k,\mu) = P_0(k) + \frac{1}{2}(3\mu^2 - 1)P_2(k)$$

- Now, the ergodic MAPS can be written as

$$\mathcal{C}_\ell^{\mathrm{E}}(\Delta\nu) = (r_c^2 r_c' B)^{-1} \sum_{k_\parallel} e^{ik_\parallel r_c' \Delta\nu} P(k,\mu)$$

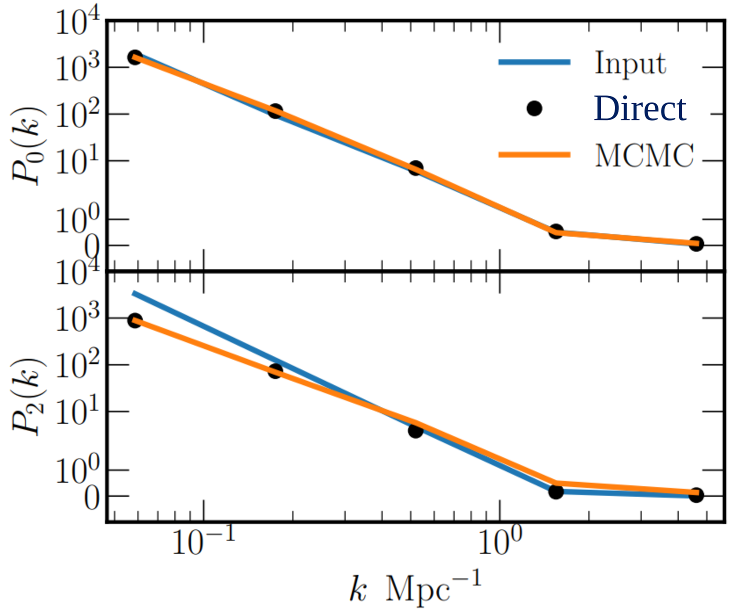
$$\mathcal{C}_\ell^{\mathrm{E}}(\Delta\nu) = \frac{1}{vfac} \sum_{k_\parallel} \mathrm{AM}(\mathrm{k}_\parallel, \Delta\nu) \times \left[P_0(k) + \frac{1}{2}(3\mu^2 - 1)P_2(k)\right]$$

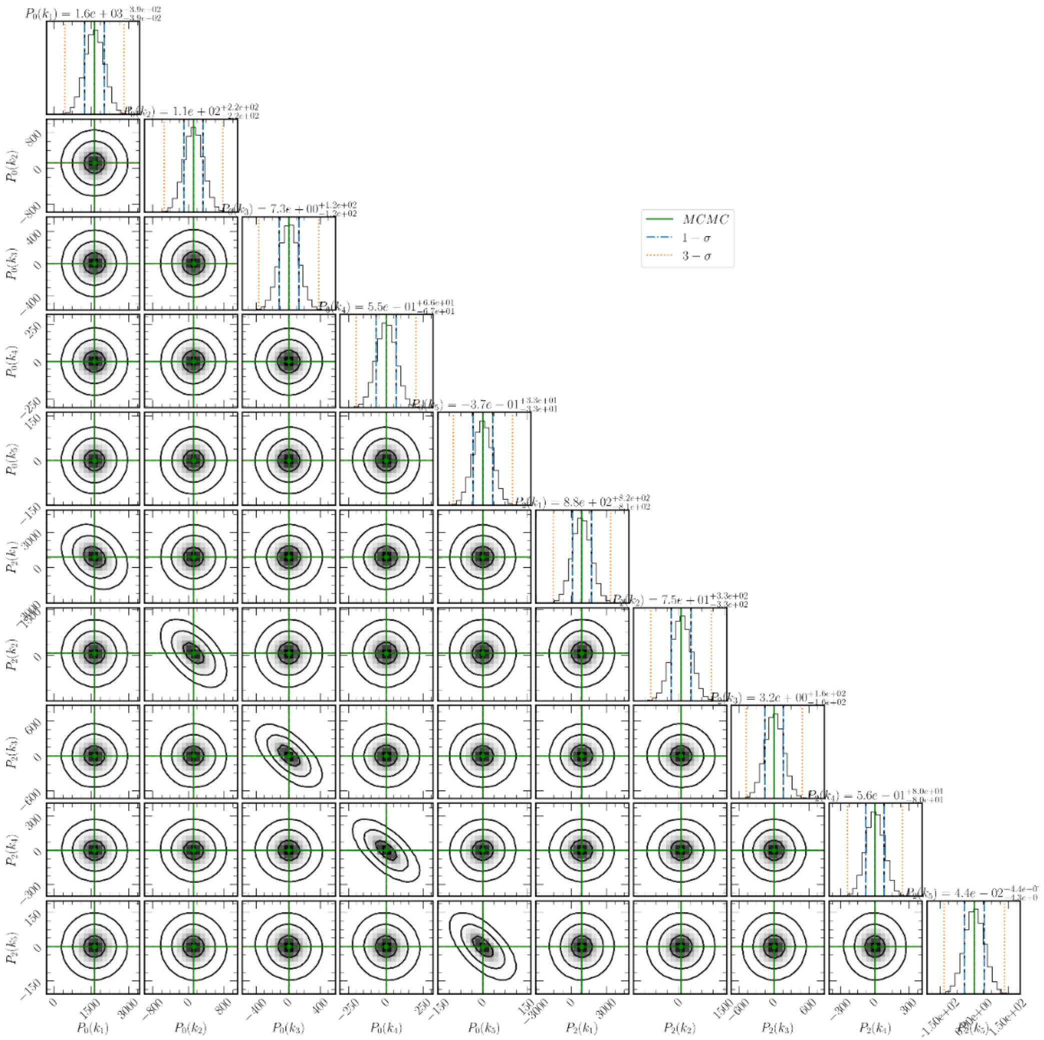
Where, $vfac = r_c^2 r_c' B$ and $\mathrm{AM}(\mathrm{k}_\parallel, \Delta\nu)$ = Fourier coefficients

# Pipeline

- We consider binned power spectrum
- These binned $P_0(k)$, $P_2(k)$ and $a$ values are the model parameters
- We can find out the maximum likelihood solution of these parameters for the data $C_\ell(\nu_1, \nu_2)$
- We use Markov Chain Monte Carlo (MCMC) to find the maximum likelihood solution

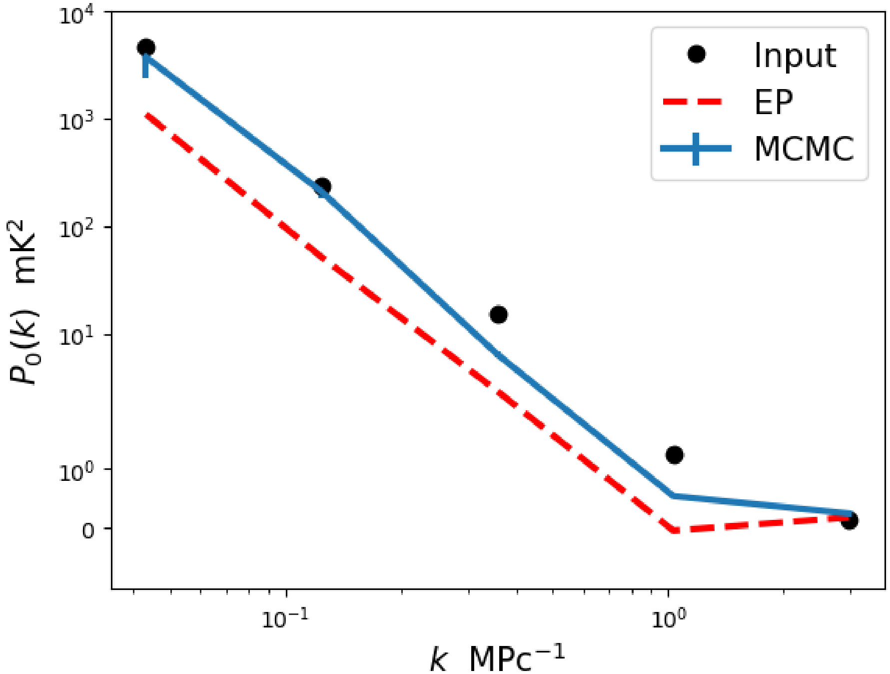
# Validating the method: Ergodic part

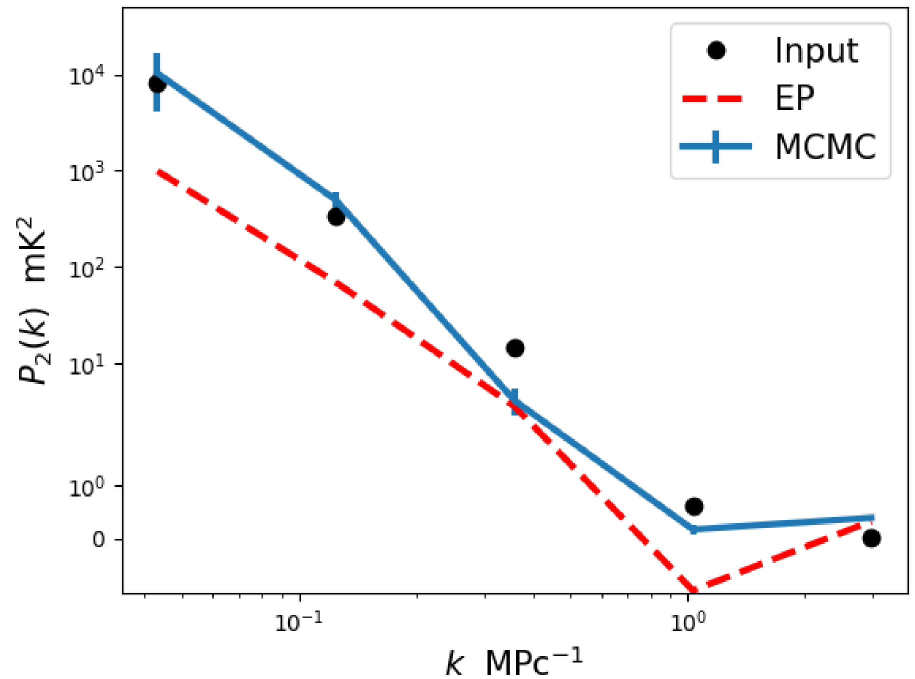


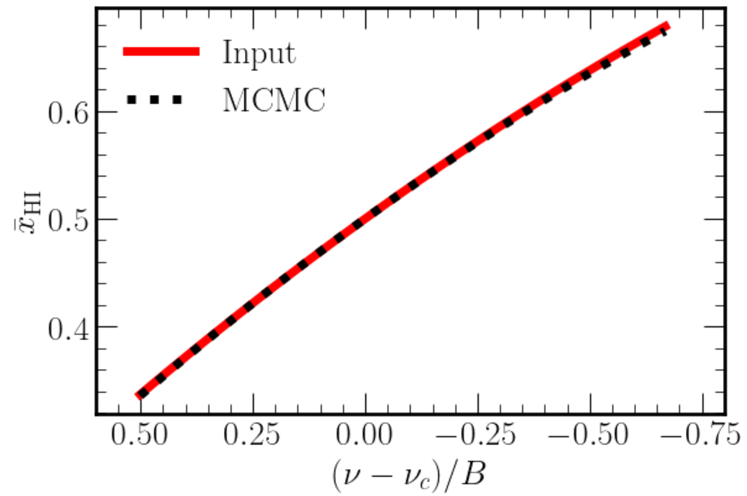




- We use 100 realizations of ergodic GRFs, simulated using known anisotropic PS.
- $P_0(k)$ and $P_2(k)$ are matching with input, this validates the modeling of ergodic part.

# Validating the method: Full model



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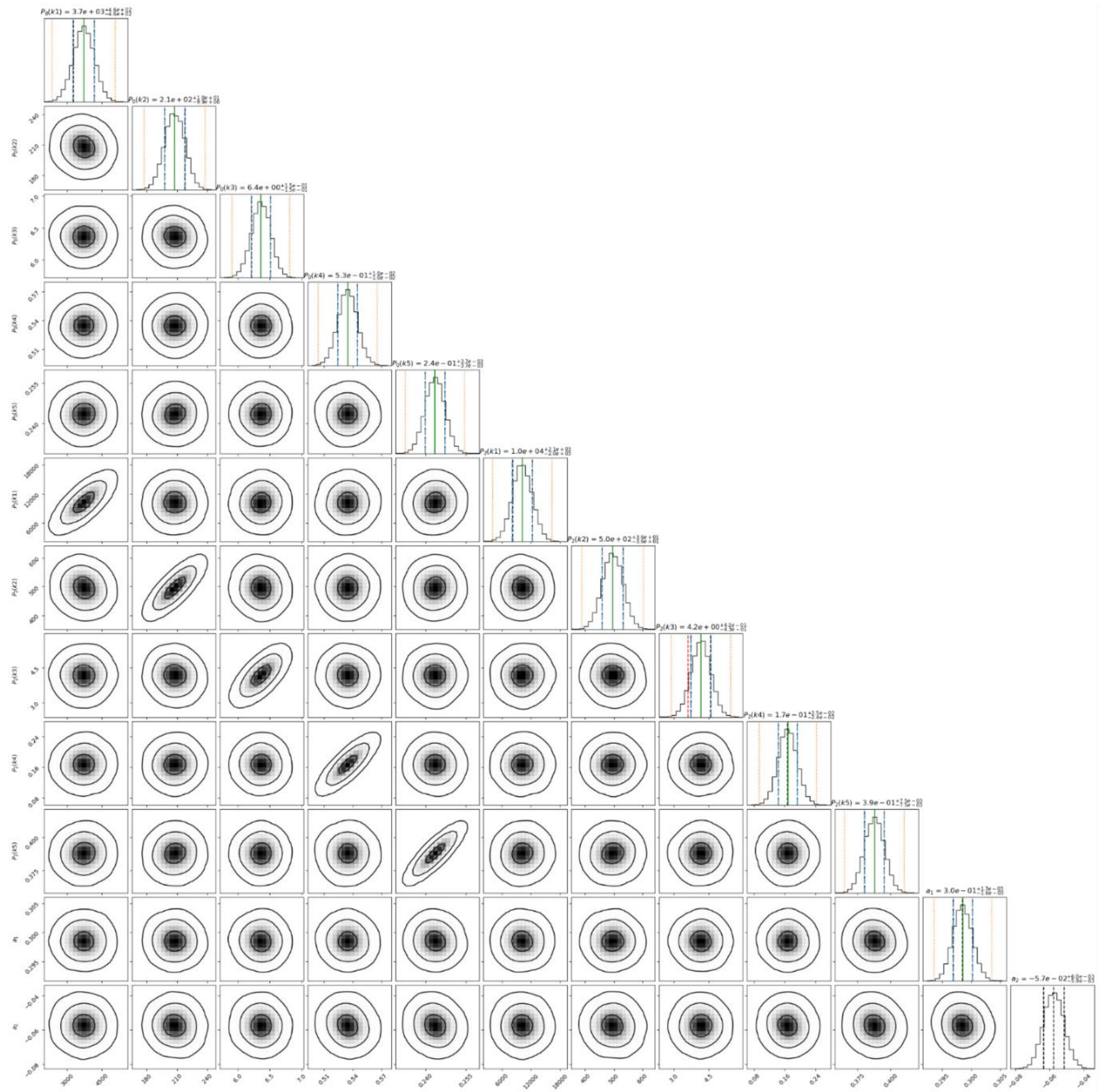
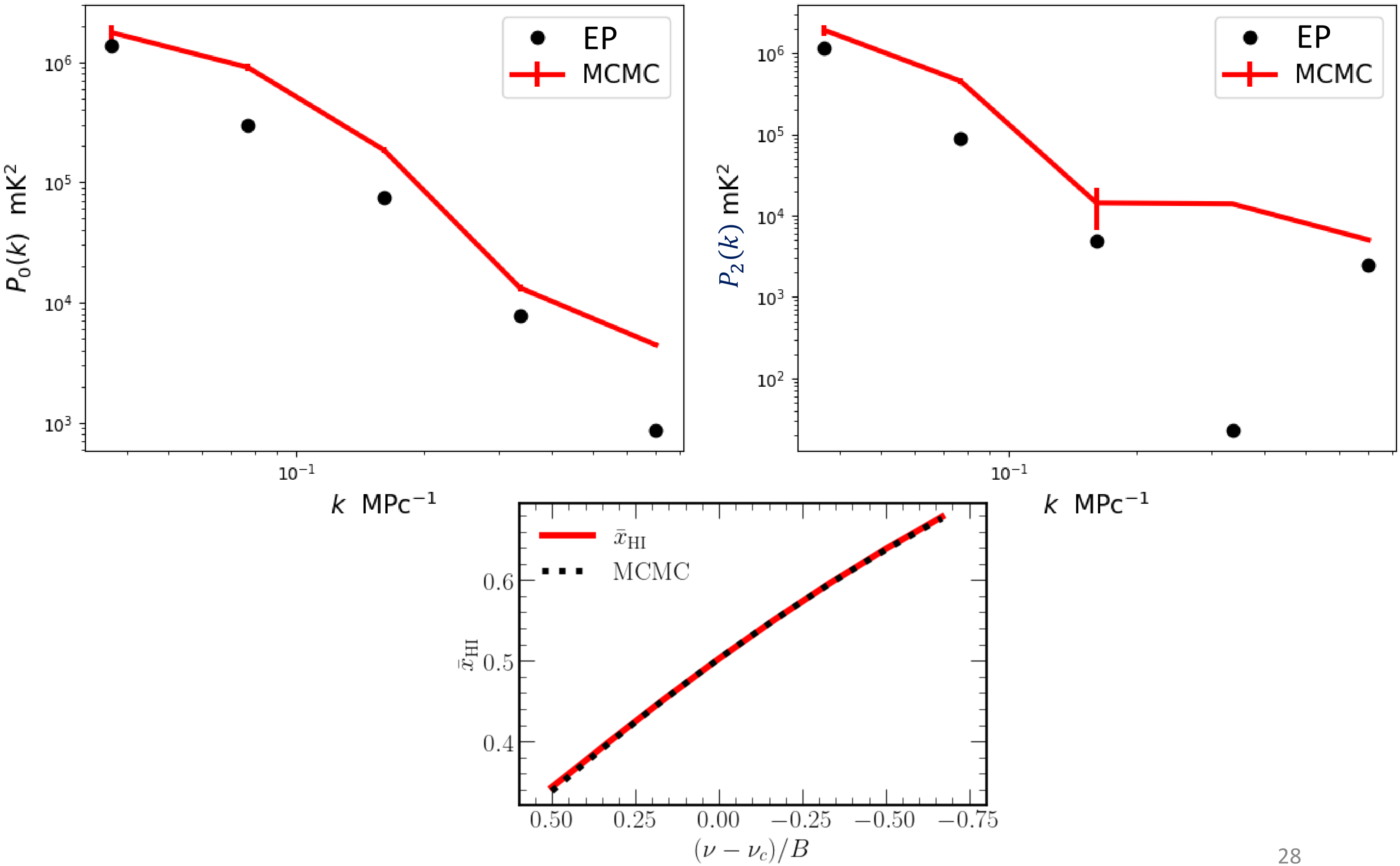


# Results for Lightcone



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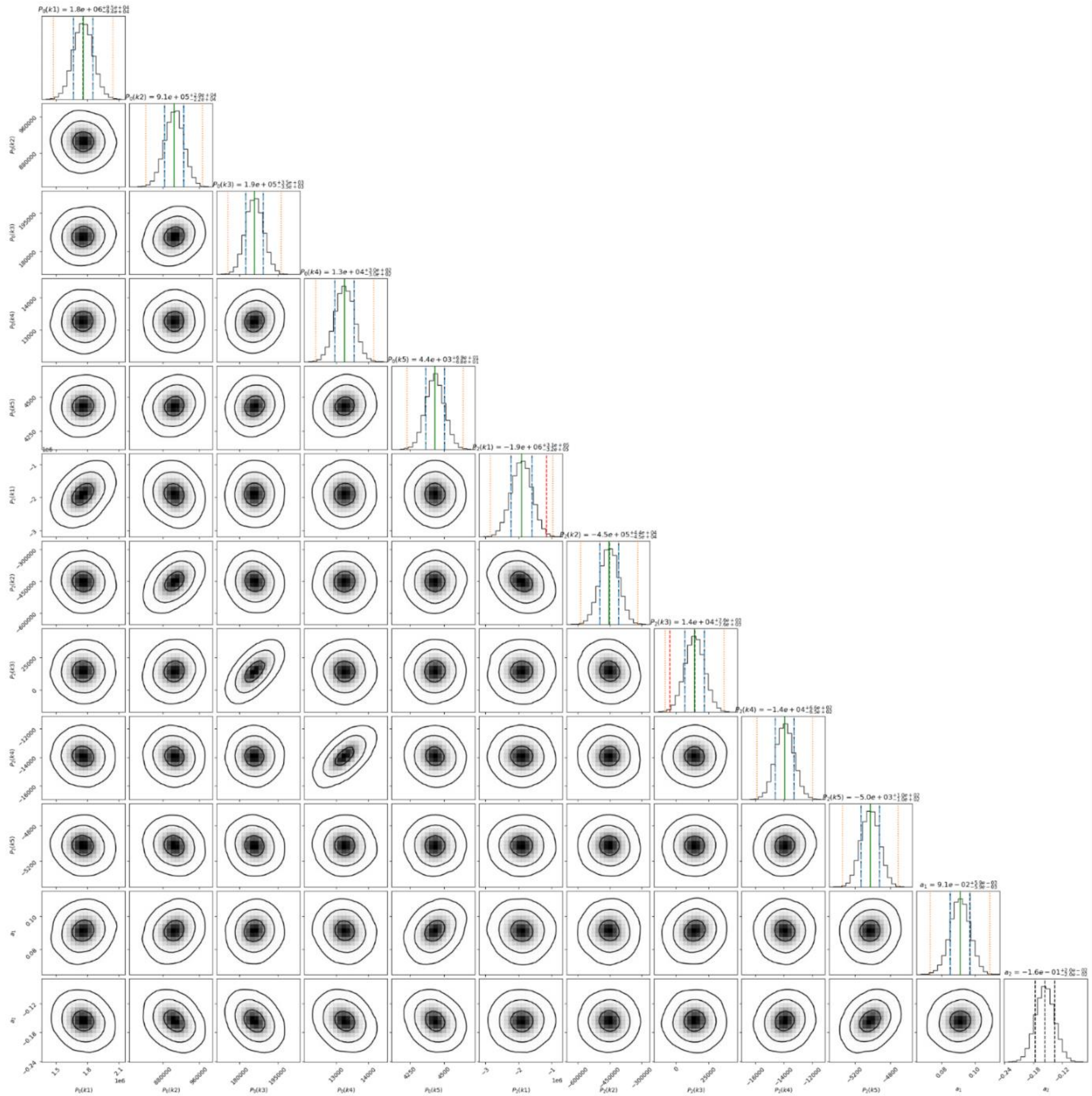


# Summary

- Real observations will have the signature for both RSD and LC effects in the $C_\ell(\nu_1, \nu_2)$ data.

- Our model can separate the evolution of $\bar{x}_{\rm HI}$ and power spectrum from the data.

- Our model can estimate the multipole components of anisotropic power spectrum.

- Direct estimation of power spectrum assume statistical homogeneity which cannot be justified in the presence of LC effect.

- We can simultaneously determine the reionization history and power spectrum multipoles from real data.


Photo Credit: Ruobing Dong

# Backup slides

# Simulating the Light-cone 21-cm signal from EoR

- For the 21-cm radiation originated from the point $\boldsymbol{n}r$, the cosmological expansion and the radial component of HI peculiar velocity $\boldsymbol{n}\,.\,\boldsymbol{v}\,(\boldsymbol{n}r,\eta)$ together determine the frequency $\nu$ at which the signal is observed, and we have

$$\nu = a(\eta)\left[1 - \boldsymbol{n}\,.\frac{\boldsymbol{v}(\boldsymbol{n}r,\eta)}{c}\right] \times \nu_e$$

- Assuming that spin temperature is much greater than the background CMB temperature, i.e $T_s \gg T_\gamma$, the HI 21-cm brightness temperature can be expressed as

$$T_b(\boldsymbol{n},\nu) = T_0 \frac{\rho_{HI}}{\rho_H}\left(\frac{H_0\nu_e}{c}\right)\left|\frac{\partial r}{\partial \nu}\right|$$

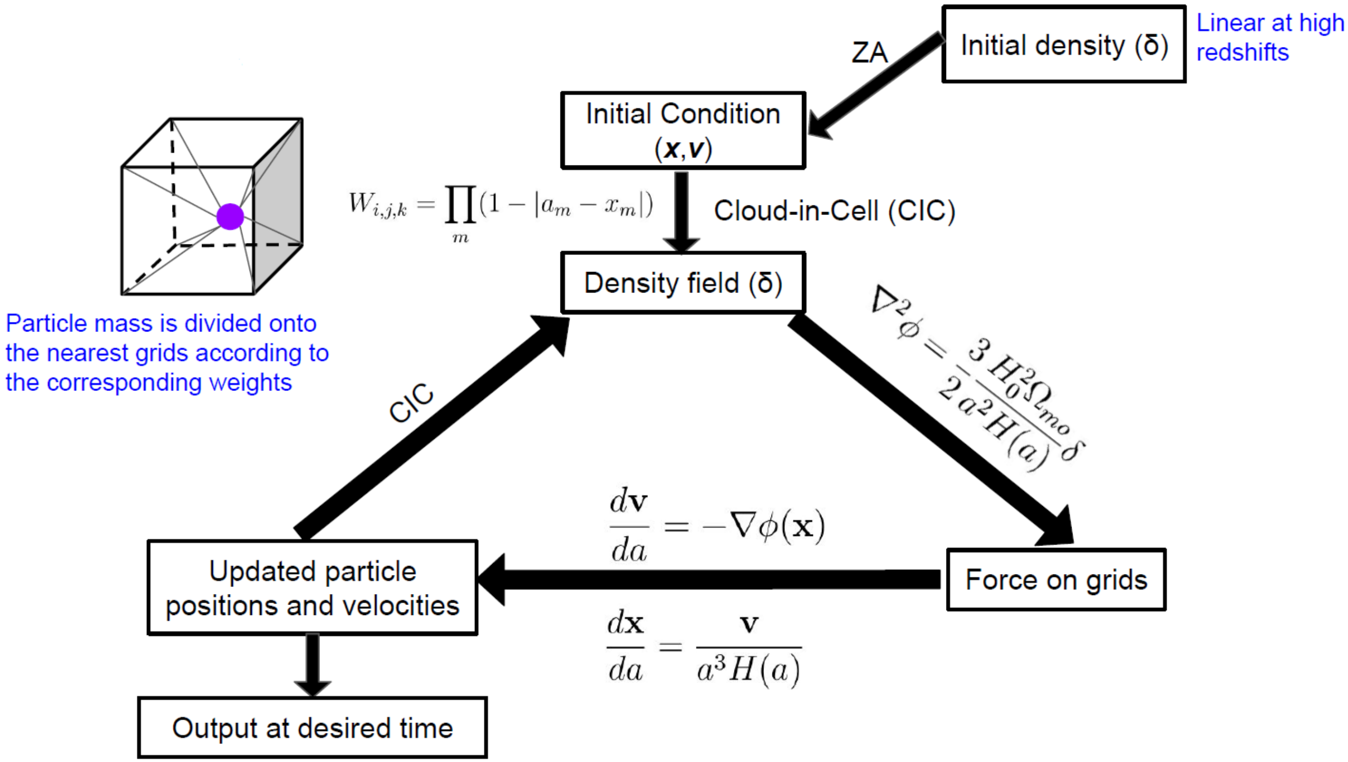
Where, $T_0 = 4.0\ mK\ \left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{0.7}{h}\right)$

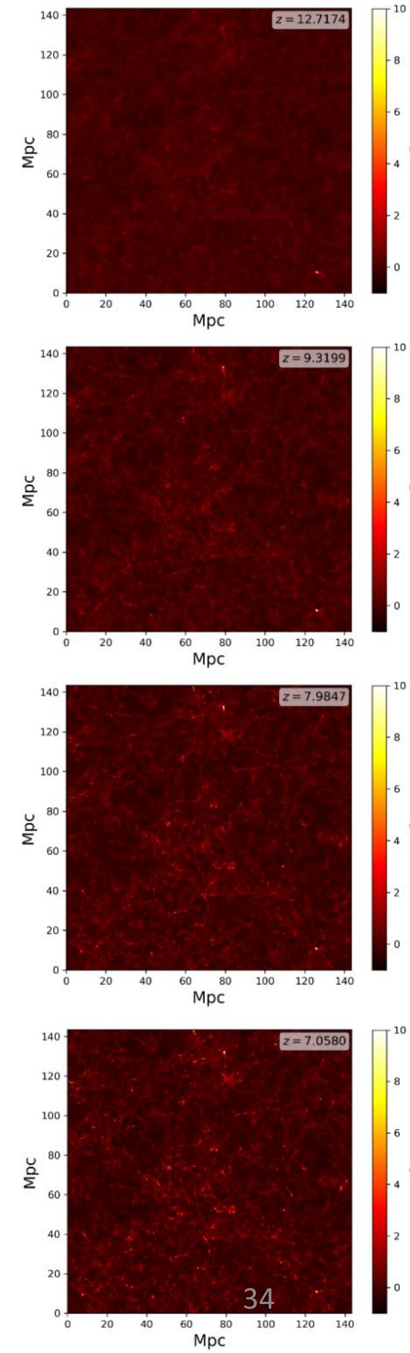
- The comoving HI density can be obtained by assigning the HI mass in the particles to a uniform rectangular grid in comoving space $\rho_{HI} = (\Delta r)^{-3}\sum_m [M_{HI}]_m$ where $(\Delta r)^3$ is the volume of each grid cell
- Here, we use a uniform grid in solid angle $(\Delta\Omega)$ and frequency $(\Delta\nu)$ to define a modified density

$$\rho'_{HI} = (\Delta\Omega\ \Delta\nu)^{-1}\left(\frac{H_0\nu_e}{c}\right)\sum_m \frac{[M_{HI}]_m}{r_m^2}$$

- Then we can calculate the brightness temperature using $T_b(\boldsymbol{n},\ \nu) = T_0\frac{\rho'_{HI}}{\rho_H}$

# Steps of Particle mesh N-body Code

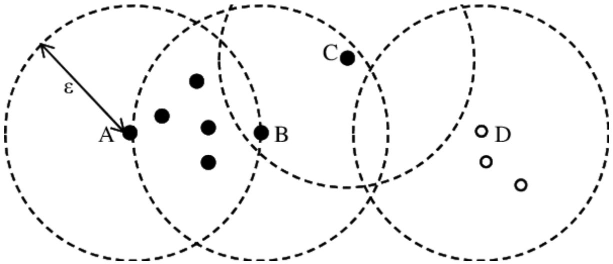


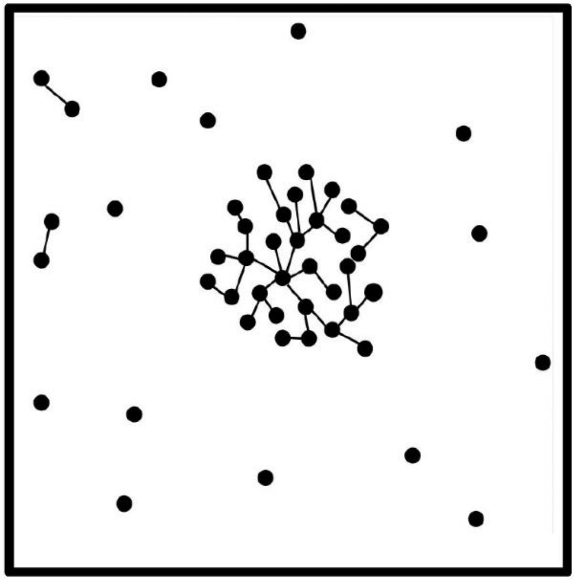



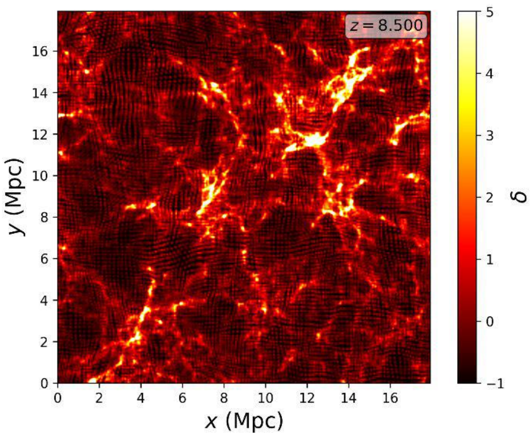

- Grid = 4096
- Grid spacing = 70 Kpc
- Number of particles = $8.58 \times 10^9$
- Box size = $(286.7 \text{ Mpc})^3$

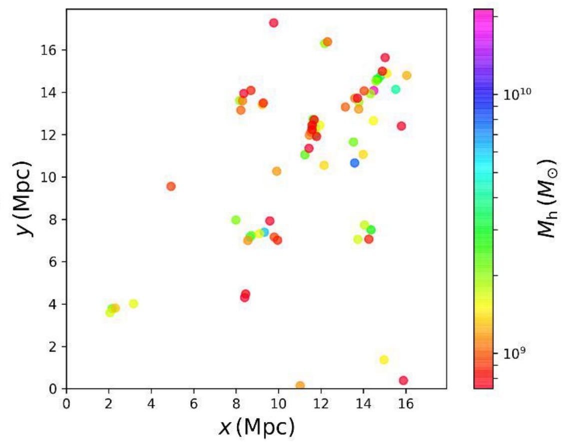
# Steps of FoF halo finder

- DM are represented by discrete particles of mass = $1.09 \times 10^8 M_{sun}$
- Group of DM particles (gravitationally bound within close vicinity) forms halo
- We look for such group of DM particles within *N*-body outputs using Friends-of-Friends (FoF) algorithm
- We call a particle friend of another particle if they are within a certain distance which is **Fixed linking length** $L_{fof} = 0.2 \times$ Grid separation = 14 Kpc
- We call a grop halo if it contains more than a minimum number of DM particles $M_{min}$ (Which is a parameter of the EoR model)



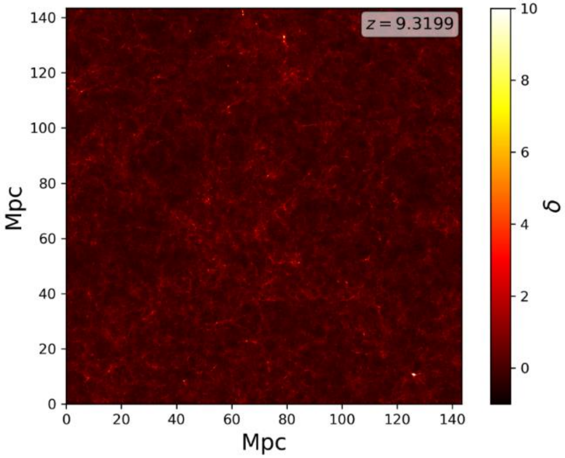


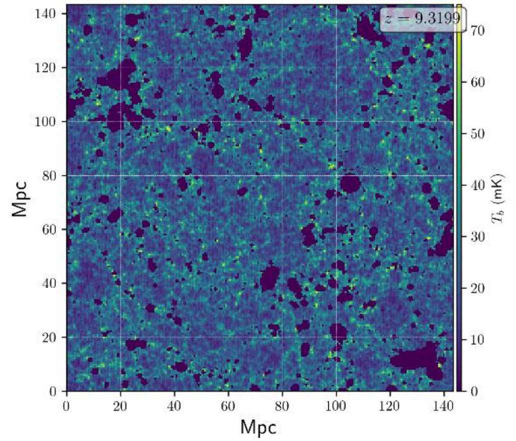



# Simulating EoR 21-cm signal

- From the N-body simulation we have DM density field, we assume that baryon will follow DM with some bias, these creates the neutral hydrogen (HI) field.
- From FoF we have dark matter halo locations and their masses.
- We illuminate those DM halo in proportion to their mass.
- So in the DM halo locations we have High photon number density ($N_\gamma$), then we smooth them using convolution with spherical tophat function.
- The smoothing radius vary from $R_{min} = grid\ size = 0.7$ Mpc to $R_{mfp}$
- $R_{mfp}$ is the mean free path of the ionizing photon through IGM.
- We then compare the photon number ($N_\gamma$) and neutral hydrogen number ($N_H$) on a grid.
- A grid is fully ionized ($x_{HII} = 1$) is $N_\gamma > N_H$
- If $N_\gamma < N_H$ then $x_{HII} = \frac{N_\gamma}{N_H}$
- Main observable of EoR is the differential brightness temperature, which can be calculated using (Bharadwaj & Ali 2005)

$$\delta T_b(\boldsymbol{x}) = 27\, x_{HI}(z,\boldsymbol{x})[1+\delta_B(z,\boldsymbol{x})]\left(\frac{H}{\frac{dv_r}{dr}+H}\right)\left(\frac{\Omega_B h^2}{0.023}\right)\left(\frac{0.15}{\Omega_m h^2}\frac{1+z}{10}\right)^{\frac{1}{2}}\left[1-\frac{T_\gamma(z)}{T_S(z,\boldsymbol{x})}\right] mK$$



# Redshift Space Distortion

- In the sky we can only observe the angular position ($\boldsymbol{\theta}$) of a source.
- If the source emits a known emission line (like 21-cm line from neutral H) then from the spectral-shift of that line one can calculate its redshift ($z = \frac{\nu_{em}}{\nu_{obs}} - 1$).
- From $z$ we calculate the commoving distance of the source using best available cosmological model

$$r_z = c\int_a^1 \frac{da}{aH(a)} \text{ where, } a = \frac{1}{1+z}$$

- If the source have some local velocity (peculiar velocity), apart from the Hubble flow then that will add-up in the redshift measurement
- Redshift-space distance $\boldsymbol{s}$ can be calculated and comes out as

$$\boldsymbol{s} = \boldsymbol{r} + \frac{\boldsymbol{v}_p.\boldsymbol{n}}{aH(a)}$$

Where, $\boldsymbol{v}_p$ is the peculiar velocity and $\boldsymbol{n}$ is the line of sight direction, $a$ is the scale factor and $H(a)$ is the Hubble parameter

# Multifrequency Angular Power Spectrum (MAPS)

- Here we decompose brightness temperature fluctuations $\delta T_b(\hat{\boldsymbol{n}},\ \nu)$ in terms of spherical harmonics $Y_\ell^m(\hat{\boldsymbol{n}})$ using

$$\delta T_b(\hat{\boldsymbol{n}}, \nu) = \sum_{\ell,m} a_{\ell m}(\nu)\, Y_\ell^m(\hat{\boldsymbol{n}})$$

- And define MAPS as

$$C_\ell(\nu_1, \nu_2) = < a_{\ell m}(\nu_1) a^*_{\ell m}(\nu_2) >$$

- In this work, it suffices to adopt the flat-sky approximation where we decompose the $\theta$ dependence of $\delta T_b(\theta, \nu)$ into 2D Fourier modes $\tilde{T}_{b2}(\boldsymbol{U}, \nu)$. Here, $\boldsymbol{U}$ is the Fourier conjugate of $\theta$, and we define the MAPS using

$$C_\ell(\nu_1, \nu_2) = C_{2\pi U}(\nu_1, \nu_2) = \Omega^{-1} < \tilde{T}_{b2}(\boldsymbol{U}, \nu)\, \tilde{T}_{b2}(-\boldsymbol{U}, \nu) >$$

Where $\Omega$ is the solid angle subtended by the simulation at the observer.