

Solution of a Hamilton-Jacobi equation using the Hopf-Lax formula

Abhishek Das

The initial value problem for the Hamilton-Jacobi equation

$$\begin{aligned} u_t + H(u_x) &= 0 \text{ in } \mathbb{R} \times (0, \infty), \\ u|_{t=0} &= g \text{ in } \mathbb{R} \end{aligned} \quad (1)$$

is solved through the minimization problem

$$u(x, t) = \min \left\{ \int_0^t L(w'(\tau)) d\tau + g(w(0)) : w \in C^1[0, t], w(t) = x \right\}, \quad (2)$$

where H and L are connected through the relation

$$H = L^* : v \longrightarrow \sup_{p \in \mathbb{R}} \{vp - L(p)\}, \quad v \in \mathbb{R}.$$

In this talk, we will discuss how appropriate hypotheses on L and g imply the well-definedness of H and that the minimization problem (2) will solve (1).