

Title: On inhomogeneous extension of Thue-Roth's type inequality with moving targets.

Abstract: For a real number x , let $\|x\|$ denote the distance of x from its nearest integer. Hardy asked the following question: *In what circumstances it is true that $\|\lambda\alpha^n\| \rightarrow 0$ as $n \rightarrow \infty$?* This question is still open in general. For example, the growth of the sequence $\|(3/2)^n\|$ is intricately linked to the famous Waring's problem. This was the motivation for Mahler to prove that for any non-integral rational number $\alpha > 1$ and any real number c with $0 < c < 1$, the inequality $\|\alpha^n\| < c^n$ has only finitely many solutions in $n \in \mathbb{N}$. Mahler also asked the characterization of all algebraic numbers having the same property as that of the non-integral rational numbers. This was answered completely by Corvaja and Zannier. In this talk, I shall present my recent work, which is an inhomogeneous analogue of Corvaja and Zannier's result. More precisely, the following: let $\alpha > 1$ be a real number. Let β be an algebraic irrational and λ be a non-zero real algebraic number. For given $\varepsilon > 0$, if there are infinitely many natural numbers n for which $\|\lambda\alpha^n + \beta\| < 2^{-\varepsilon n}$ holds true, then α is transcendental. I shall also discuss some other related works about simultaneous approximation of algebraic numbers and the trace of powers of algebraic numbers.